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FORMAL ONTOLOGY, PROPOSITIONAL IDENTITY AND TRUTH

In the philosophical tradition, propositions have a double nature. On one hand, they are units of sense of a fundamental logical type. They are *senses of sentences* provided with *truth conditions*. Each proposition is *true in a circumstance* when it represents a fact which exists in that circumstance. So any analysis of the logical form of propositions requires a theory of truth. On the other hand, propositions are also the *contents* of *conceptual thoughts* that we, human beings, have whenever we represent facts of the world[1]. In particular, they are the contents of *illocutionary acts* like assertions, promises and questions that we perform in the use of language. They are also the contents of our *attitudes* (beliefs, desires, intentions) towards objects and facts.

As Frege pointed out, the two intrinsic aspects of propositions are logically related. For we cannot express a propositional content in thinking and speaking without relating it *eo ipso* to the world with a certain *force*[2]. Any literal meaningful utterance of an elementary sentence is always an attempt by the speaker to perform an illocutionary act with a force F and a propositional content P[3]. So *force, sense and denotation* are the three basic components of sentence meaning in the logical structure of language. And the proposition which is the sense of an elementary sentence in a context of utterance is also the content of the elementary illocutionary act that the speaker of that context would mean to perform if he or she were using that single sentence literally. The main purpose of this chapter is to formulate a **new theory of sense and denotation** that takes into account the double nature of propositions. The fact that propositions are contents of conceptual thoughts imposes to propositional logic many conditions of material and formal adequacy that logicians have unfortunately tended to neglect until now. We, human agents, have cognitive abilities which are both restricted and creative. We can only utter a finite number of sentences and make a finite number of acts of reference and predication in a context of utterance. So we can only have in mind a finite number of propositions with a finite structure of constituents. However we are free and creative. We can understand infinitely many sentences, utter new sentences and have new thoughts. We are neither omniscient nor perfectly rational. We understand most propositions without knowing whether they are true or false. We often make false assertions and sometimes believe necessarily false propositions. But we are always minimally consistent in a sense that remains to be explained. Propositional logic has to account for such facts.

In particular, the theory of truth for propositions must be compatible with the theory of success and satisfaction for illocutionary acts. By nature, illocutionary acts have *felicity conditions* (Austin, 1956). Attempts to perform illocutionary acts can *succeed* or *fail*. In order to succeed to perform an illocutionary act a speaker must make a right attempt in a right context. Moreover illocutionary acts are directed at objects and facts in the world. Speakers in general attempt to achieve a success of fit between words and things. Their illocutionary acts are *satisfied* only if represented facts turn out to be existent. In particular, assertions are satisfied when they are true, promises are satisfied when they are kept and directives when they are obeyed. As Searle and I pointed out[4], there is no way to elaborate an adequate theory of success and satisfaction for illocutionary acts without identifying their contents with propositions. So the formal ontology of illocutionary logic is realist. Moreover success, satisfaction and truth are logically related. So we need a unified theory of force, sense and denotation[5].

Unfortunately, current philosophical logics of sense and denotation are incompatible with an adequate analysis of thought. Standard modal, temporal, epistemic, intensional and agentive logics are based on Carnap's definition of the logical type of propositions. They tend to reduce propositions to their truth conditions. Such a reduction is incompatible with contemporary philosophy of language, mind and action. From a philosophical point of view, *strict equivalence* (the property of having the same truth values in the same circumstances) is not a sufficient criterion of propositional identity. Many speech acts with the same illocutionary force and strictly equivalent propositional contents have different success and sincerity conditions. Thus the assertion (or belief) that Ottawa is the capital of Canada is different from the assertion (or belief) that Ottawa is the capital of Canada and not a real number.

I will formulate an analysis of the logical form of propositions which is adequate for the purpose of the theory of speaker and sentence meaning. On my view, the primary units of meaning in the use and comprehension of language are not isolated propositions but complete illocutionary acts of the form F(P) provided with felicity rather than truth conditions. So no proposition can be the sense of a sentence in a context of utterance according to a possible semantic interpretation unless it is also a possible content of illocutionary act. My notion of proposition is general; it covers senses of all types of sentence (declarative or not). The same proposition can be the common sense of sentences of different syntactic types e.g. "You will help Paul" and "Please, help Paul!" just as it can be the common content of illocutionary acts with different forces[6].

This chapter has the following content. The first section presents the formal ontology of my theory of types of sense and denotation and the second section my analysis of the logical type of proposition. The third section formulates a concise definition of truth according to predication and explicates a new relation of strong propositional implication which is important for the account of rationality. The fourth section illustrates my theory of sense; it proceeds to the analysis of modal and temporal propositions within the logic of ramified time. That section defines the ideographic object language of a rich philosophical logic. The fifth section presents the formal semantics of that logic and the sixth section an axiomatic system where valid laws are provable. Finally, the last section states a series of important valid laws governing truth, propositional identity and strong implication. It shows striking differences existing between my logic of sense and current modal and intensional logics of Lewis, Carnap and Montague, Hintikka's epistemic logic, Belnap's logic of relevance, Parry's logic of analytic implication and Cresswell's hyperintensional logic.

1. THEORY OF TYPES

On the basis of preceding considerations about thought and meaning, I advocate a Frege — Church formal ontology much richer than that of Russell[7]. I propose to stratify as follows *the universe of discourse in the theory of types of philosophical logic*[8]:

1. **There are three primitive types of denotation:** the type e of *individuals*, the type t of *truth values* and the type s of *success values*. Individuals are particular objects like material bodies and persons existing in actual or possible courses of the world. They are objects of reference of the simplest logical kind. There is at least one individual in the world. So there is a non empty set *Individuals* of individual objects

in the universe of discourse. The two truth values are *truth* and *falsity* and the two success values *success* and *insuccess*[9].

2. **There are two primitive types of sense:** the type c of *concepts of individuals* and the type r of *attributes* of individuals. *Properties* of *individuals* like being alive and, for each number $n \geq 2$, *relations of degree n between individuals* like being taller than are attributes of individuals. In my symbolism, Concepts is the set of individual concepts and Attributes the set of attributes of individuals. So the set of primitive senses is the union *Concepts* $\cup$ *Attributes*.

As Frege and Church pointed out, *there is a fundamental relation of correspondence between senses and denotations* in the universe of discourse. Actual denotations of certain types correspond to senses in possible circumstances[10]. Thus *propositions*, which are *senses of sentences*, have *truth values* as denotations; they are either true or false in each circumstance. Concepts of individual objects, which are senses of referring expressions like the king of France, have single individuals as denotations: they apply to at most one individual object in each circumstance. The denotation corresponding to a sense can of course vary from one circumstance to another. In the logic of ramified time and of action, each *circumstance* is a pair m/h of a moment of time m and of a history h to which that moment belongs. Different persons have been king of France in the past. In certain circumstances no individual falls under a concept. There is no actual king of France at the present moment. There is a next king of France in a circumstance m/h when someone becomes king at a moment m' posterior to m in the history h.

Properties of individual objects, which are senses of unary predicates, have sets of individuals under concepts as denotations: a certain number of *individuals under concepts* possess them in each circumstance. Relations of degree n between individuals are senses of n-ary predicates where $n \geq 2$. In thinking and speaking, we predicate relations of degree n in a certain order. We apply them successively to n objects of reference. The order of predication is expressed in language use by the syntaxical order in which referential expressions occur in atomic clauses. In thinking that Goliath is taller than David we predicate the relation of being taller first to David next to Goliath. Most binary relations between individuals are not symmetric. So these relations are satisfied by individuals under concepts in a certain order. Goliath is taller than David but David is not taller than Goliath. This is why attributes are said to be satisfied by sequences in the calculus of predicates[11].

As Frege [1892] pointed out, the truth value of many propositions depends on the sense rather than the denotation of their propositional constituents. Certain attributes of individuals are *intensional*: they are satisfied by sequences of individuals under some concepts without being satisfied by the same sequences of individuals under other concepts. Thus the relation of desire is intensional: Oedipus desired to marry Jocasta, the Queen of Thebes; but he did not desire to marry his mother. For that reason, denotations of attributes are sequences of individual concepts (or of individuals under concepts) rather than sequences of pure individuals in philosophical logic[12]. There are however *extensional* attributes whose denotation only depends on the denotations of the concepts which satisfy them. So are the property of being tall and the relation of moving something. If Jocasta, the queen of Thebes is tall so is Oedipus' mother. *Extensional attributes* are satisfied by a sequence of individuals under concepts in a circumstance when they are satisfied by the sequence of individuals that fall under these concepts in that circumstance. For the sake of simplicity, I will often give examples of extensional attributes.

3. **Each type is a subtype of more general types.** For any pair of types α and β of entities of the universe of discourse, there is the derived type $\alpha \cup \beta$ of all entities which are of the type α or β. By definition, the set $U_{\alpha \cup \beta}$ of entities of type $\alpha \cup \beta$ is the union $U_\alpha \cup U_\beta$ of the set U_α of entities of type α and of the set U_β of entities of type β. Thus concepts and attributes have the more general type of *propositional constituents*[13]. For example, $c \cup r$ is the type of propositional constituents of first order propositions about individual objects[14].

As in intensional logic, the set of types of entities is closed under the following two other operations:

4. For any pair of types α and β, **there is the derived type $(\alpha\beta)$ of *functions* from the set of all entities of type α into the set of all entities of type β.** By definition, the set $U_{\alpha\beta}$ of entities of type $(\alpha\beta)$ is the set of functions $U_\alpha \rightarrow U_\beta$. Thus (tt) is the type of unary truth functions and t(tt) that of binary truth functions. (et) is the type of (characteristic function of) sets of individuals and e(et) that of sets of pairs of individuals. As usual, sets of n-ary sequences of entities of the type α are of the type $(\alpha(\ldots(\alpha t)\ldots))$ with n left and n right parentheses.

5. Finally, for any type $\#\alpha$ of entities, **there is the derived type $\#\alpha$ of intensions whose extensions are entities of type α.** An

intension[15] of type $\#\alpha$ is a function from the set *Circumstances* of all possible circumstances into the set of entities of type α. Thus the set $U_{\#\alpha}$ of entities of type $\#\alpha$ is the set of functions $Circumstances \rightarrow U_{\alpha}$. For example, Carnapian truth conditions are intensions of type $\#t$: they are functions which associate with any possible circumstance a single truth value.

All types of first order senses and denotations of the universe of discourse can be obtained from the few primitive types named above by applying the three operations on types that I have defined. From Carnap we know that ***each sense to which correspond entities of type α has a characteristic intension of type $\#\alpha$***, namely the function which associates with any possible circumstance the actual entity which is the denotation of that sense in that very circumstance. So any proposition has its characteristic Carnapian truth conditions which associate with each possible circumstance the true if and only if that proposition is true in that circumstance. Unfortunately traditional intensional logic has tended to identify senses with their characteristic intension. So propositions are reduced to truth conditions: their type p is $\#t$ in the modal logic of Carnap, Prior, Montague, Kaplan, Kripke, Belnap and most other logicians. *Strictly equivalent* propositions, which are true in the same possible circumstances, are then identified. There is only one necessarily true proposition as well as only one necessarily false proposition according to current philosophical logic.

However, it is clear that most strictly equivalent propositions do not have the same cognitive values. In particular, they are not substituable *salva felicitate* within the scope of illocutionary forces and psychological modes. For example, one can assert (and believe) that a big city is a city without asserting (or believing) *eo ipso* that $\sqrt{2}$ is an irrational number, even if these two assertions (and beliefs) are both true in all possible circumstances. Philosophy of language and mind requires a much finer logic of sense. Just as the same denotation can correspond to different senses, the same intension can be common to different senses in the deep logical structure of language.

2. THE LOGICAL FORM OF PROPOSITIONS

In order to seriously take into account the fact that propositions are always expressed in the attempted performance of illocutionary acts, I have advocated in ***Meaning and Speech Acts*** and other papers[16] a *natural predicative logic of propositions.* My main idea was to explicate the logical type of proposition by mainly taking into consideration the acts of predication that we make in expressing and understanding propositions. My propositional logic according to predication is based on the following principles:

2.1. Propositional constituents are senses and not pure denotations

As Frege (1892) pointed out, we cannot *refer to* objects without subsuming them under senses and without *predicating of* them attributes. Thus referential and predicative expressions of sentences have a sense in addition to a possible denotation in each possible context of utterance. When we speak literally. we conceive the concepts and attributes which are the senses of the referential and predicative expressions that we use. Moreover we refer to the objects which fall under these concepts in the context of utterance. Frege's argument against direct reference is valid if propositions are contents of thought. Otherwise, we would be totally inconsistent. We can make mistakes and assert, for example, that Hesperus is not Phosphorus[17]. But we never intend to make the absurd assertion that Hesperus is not Hesperus. Consequently, there are no *singular propositions* whose constituents are pure individual objects in my formal ontology, contrary to what Russell and others advocate in defending direct reference or externalism. All our objects of reference are *objects under concepts.*

2.2. Propositions have a structure of constituents

In speaking and thinking conceptually, we always predicate in a certain order attributes of our objects of reference. Just as any clause of an elementary sentence is composed from one or several atomic clauses where predicates of degree n are syntactically combined in a certain order with a number n of complete referential expressions, each propositional content that we have in mind is composed from one or several atomic propositions whose expression consists in an act of predication. To express an atomic proposition is just to predicate in a certain order an attribute of degree n of n individual objects under concepts[18]. So each atomic proposition has a number $n + 1$ of propositional constituents. It contains a main attribute of individuals R_n of degree n and

n individual concepts $u_c^1, \ldots, u_c^n$. And it is true in a circumstance when the sequence corresponding to the order of predication of its n individuals under concepts belongs to the actual denotation of its attribute in that circumstance. The order of predication is important only when it changes the truth conditions. The relation of identity is by nature symmetric. So the order in which we predicate the relation of identity of two entities is not important. We express the same atomic proposition in thinking that the morning star is the evening star and in thinking that the evening star is the morning star. On the contrary, the relation of admiration is not symmetric. A person can admire someone without being admired by him or her. So we express different atomic propositions when we predicate in a different order the relation of admiration of two objects of reference. We do not think that Napoleon admires Chirac when we think that Chirac admires Napoleon. Atomic propositions are identical when they have the same propositional constituents and they are true in the same possible circumstances. So atomic propositions are of the type $a = ((c \cup r)t)((\#t)t)$. Each atomic proposition ua is a pair whose first term id1(ua) is a finite non empty set of propositional constituents and whose second term id2(ua) is a set of possible circumstances. And the set U_a of atomic propositions is a special proper subset of the set $(P(Attributes) \cup P(Concepts)) \times P(Circumstances)$ in my logic of sense.

Elementary propositions like the proposition that the actual pope speaks Russian are composed from a single atomic proposition. Complex propositions like the proposition that the actual pope is Polish and speaks Russian are composed from several atomic propositions. I will say that two propositions have *the same structure of constituents* when they are composed from the same atomic propositions. In order to be identical two propositions must have the same structure of constituents.

2.3. An adequate explication of truth conditions must take into account the effective way in which we understand such conditions

To understand the truth conditions of an elementary proposition is not to know the actual truth value of its single atomic proposition in each possible circumstance. We understand the elementary proposition that the biggest whale is a fish without knowing *eo ipso* that it is necessarily false. We discovered in the course of history that whales are mammals. We understand expressed elementary propositions without knowing whether they are true or not in the context of utterance. Consider elementary propositions of the simplest kind which predicate an extensional property of an individual under concept. In understanding such elementary propositions we just understand that they are true in all (and only) the circumstances where the individual which falls under their concept possesses their characteristic property. We in general do not know by virtue of competence actual denotations of propositional constituents in the context of utterance. We often refer to an object under a concept without being able to identify the object falling under that concept. Someone who says that Julie's murderer is wounded can just refer to whoever in the world is her murderer. Our knowledge of the world is not only partial. Some of our beliefs are false. We can wrongly believe that objects of reference possess a certain property. So we can refer to objects which do not fall under the concept that we have in mind[19]. Furthermore, the objects to which we refer can possess predicated properties in many different ways. Julie's murderer could be wounded in various places.

From a cognitive point of view, **different possible denotations could then correspond according to us to an expressed attribute or concept in each circumstance. In apprehending propositional constituents we rarely identify their actual denotations. We just *presuppose* that they have one in each circumstance. However we are able to consider by virtue of competence possible denotations that these senses could have in each circumstance.** We might ignore who is Julie's murderer and not be sure that her murderer is wounded at a moment of utterance. But we can at least think of various people who could have murdered Julie and who could be wounded at that moment.

Any speaker who conceives propositional constituents can in principle assign to them possible denotations of the appropriate type in the circumstances that he or she considers. Our possible assignments of a denotation to propositional constituents associate a single individual (or no individual at all) with each concept and a unique set of individuals under concepts with each property in each possible circumstance. Let us give a few examples. According to a first possible denotation assignment, my friend Paul would be Julie's murderer and he would also be wounded at the moment m according to history h. According to a second one, the chief of police, Julius, would be Julie's murderer but he would not be wounded at all in the circumstance m/h. According to a third one, nobody would be Julie's murderer but Paul, Julius and other people would be wounded in m/h, and so on. We clearly do not know a priori which possible assignments of a denotation to such senses match the reality. But we are at least capable by virtue of competence of distinguishing denotation assignments according to which an atomic proposition is true from denotation assignments according to which it is false. Thus we know that the atomic proposition that Julie's murderer is wounded is true in the circumstance m/h according to the first possible assignment considered above and that it is false in that circumstance according to the two others.

Most atomic propositions have a lot of *possible truth conditions* in our interpretations: they could be true according to us in a lot of different sets of possible circumstances given the various denotations that their propositional constituents could have in circumstances. From a logical point of view, **there are as many possible truth conditions for an atomic proposition as there are different sets of possible circumstances where that proposition could be true.** Possible truth conditions are of the type $\#t$. An atomic proposition *could be true in a possible circumstance according to an agent* when it is true in that circumstance according to at least one way that agent could assign a denotation to its propositional constituents. Any agent who takes into consideration a number n of different possible circumstances can in principle assign 2^n different possible truth conditions to atomic propositions in his or her interpretation. **Among all possible truth conditions for an atomic proposition u_a, there are of course its *actual Carnapian truth conditions* to which correspond the set of possible circumstances $id_2(u_a)$ where it is true.**

Many atomic propositions are logically related. If Paul is Julie's husband and Julie's husband is taller than Jim, then Paul is taller than Jim. The first two atomic propositions could not be true in a circumstance unless the third one is also true in that very circumstance. Possible truth conditions of atomic propositions are logically related. **So our possible interpretations respect meaning postulates of a logical nature in assigning possible denotations to propositional constituents in circumstances. Our *possible valuations* of senses assign to atomic propositions possible truth conditions that they all could have together.**

From a logical point of view, possible valuations of senses are functions assigning to concepts and attributes denotations of the appropriate type in possible circumstances. By definition, $val(u_c)\ m/h \in$

Individuals when an individual object falls under the individual concept uc according to the possible valuation *val*. Otherwise, $val(u_c)\ m/h = \varnothing$.[20] And $val(R_n)m/h \in P(Concepts^n)$ for any attribute R_n of degree n. So the set $Val_{c \cup r}$ of possible valuations of propositional constituents is a proper subset of the set $(Concepts \rightarrow (Circumstances \rightarrow Individuals \cup \varnothing)) \cup (Attributes \rightarrow (Circumstances \rightarrow \bigcup_{1<n} P(Conceptsn))$.

Among all possible valuations of propositional constituents there is of course the real valuation (in symbols val^*) which associates with concepts and attributes their actual denotation in each possible circumstance. Thus $val^*(u_c)m/h$ is the object which really falls under individual concept uc (in case there is one) and $val^*(R_n)m/h$ the sequence of objects under concepts which really satisfy attribute R_n in the circumstance m/h. We, human agents, are not aware of actual denotations of most senses in most circumstances. So our possible valuations of senses can associate a non actual denotation to many propositional constituents in many circumstances. A possible valuation $val \in Val$ *is a real valuation of* an individual concept u_c and of an attribute R_n when, for any circumstance $m/h, val(u_c) = val^*(u_c)m, h$ and $val(R_n)m/h = val^*(R_n)m/h$.

By definition possible valuations of senses respect meaning postulates for logical attributes and operations on attributes that are imposed by their logical form. We all know by virtue of competence that individuals that we subsume under different concepts are identical in a circumstance when these concepts apply to the same individual in that circumstance. So all possible valuations val assign the same denotation to the identity relation = between individuals which is a logical universal. According to any interpretation a pair of individuals under concepts $u_c^1, u_c^2 \in val(=)m/h$ if and only if $val(u_c^1)m/h = val(u_c^2)m/h$. Individual concepts have internal properties. So any object of reference which falls according to a valuation under certain concepts (e. g. the concept of being Oedipus' mother) in a circumstance possesses according to that valuation essential properties (e. g. to be a woman) in all circumstances where it is existent. So certain atomic propositions (e. g., that Oedipus' mother is male) are contradictory: they are false in all circumstances according to every possible valuation of an interpretation. Furthermore, objects of reference which possess certain properties (e. g. to be fallible) in a circumstance according to certain valuations must have other properties (e. g. to make a mistake) in the same or other possible circumstances.

As one can expect, each possible valuation of propositional constituents associates *particular possible truth conditions* with all atomic propositions containing these constituents. By virtue of its logical form, an atomic proposition $R_n(u_c^1, \ldots, u_c^n)$ predicating an attribute R_n of n individuals under concepts $u_c^1, \ldots, u_c^n$ in that order *is true according to a valuation val in a circumstance m/h* if and only if $< u_c^1, \ldots, u_c^n > \in val(R_n)m/h$[21]. **So to each possible valuation of propositional constituents corresponds a unique possible valuation associating with all atomic propositions possible truth conditions that they could all have together. Possible valuations val of atomic propositions are of the type $a\#t$. They belong to a proper subset *Vala* of the set of functions $U_a \rightarrow$ Pcircumstances.** Each possible valuation of atomic propositions $val+ \in Val_a$ is the extension of a possible valuation of senses $val \in Val_{cUr}$ such that $m/h \in val + (R_n(u_c^1, \ldots, u_c^n))$ if and only if $< u_c^1, \ldots, u_c^n > \in val\ (R_n)m/h$. **Consequently, the distinguished real valuation of senses val*, which assigns to all attributes and concepts their actual denotation in each circumstance, determines the real valuation val^*+ of atomic propositions that associates with each of them its actual Carnapian truth conditions in the reality.** By definition, an atomic proposition u_a *is true at a moment m according to a history h* if and only if $m/h \in val^* + (u_a)$. For $val^* + (u_a)$ is by definition $id_2(u_a)$.

We *a priori* know actual truth values of few propositions, just as we a priori know the actual denotations of few propositional constituents. Firstly, there are few tautological (or contradictory) atomic propositions that we *a priori* know to be necessarily true (or false) by virtue of competence. In my terminology, *tautological* atomic propositions predicate of a sequence of objects of reference an attribute that we *a priori* know that they satisfy (e. g. that Platon's mother is a woman) given the logical form of these concepts and of that attribute. Their only possible truth condition in our mind is then the set of all possible circumstances. On the contrary, *contradictory* atomic propositions predicate of a sequence of objects of reference an attribute that we *a priori* know that they do not satisfy (e. g. that Platon is different from himself). Their only possible truth condition in our interpretations is the empty set of all possible circumstances. Elementary propositions containing a tautological (or contradictory) atomic proposition are exceptions. They are necessarily and analytically true (or false). Most elementary propositions are contingently, *a posteriori* and synthetically true or false. We need to observe the world in order to know whether they are true or false.

Moreover, the truth of most complex propositions containing several atomic propositions is compatible with various possible ways in which objects could be. Think of disjunctions, past and future propositions, historic possibilities, etc. Consider the past proposition that the pope was sick. In order that it be true in a given circumstance, it is sufficient that the pope be sick in at least one previous circumstance. So the truth of that past proposition in any circumstance c is compatible with a lot of possible truth conditions of the atomic proposition attributing to the pope the property of being sick[22].

When a proposition contains several atomic propositions, its truth in a circumstance is compatible with certain possible valuations of its atomic propositions and incompatible with all others[23]. We can ignore actual truth conditions. But we always distinguish, when we have a propositional content in mind, the possible valuations of its atomic propositions which are compatible with its truth in a possible circumstance, from those which are not. In making such a distinction our mind draws a kind of ***truth table***. Thus we know that the truth of an elementary proposition in any circumstance is compatible with all and only the possible valuations of its single atomic proposition according to which it is true in that very circumstance. We know that the truth of a propositional negation $\neg P$ in a circumstance is compatible with all and only the possible valuations of its atomic propositions which are incompatible with the truth of P in that circumstance. And that the truth of the modal proposition that it is universally necessary that P is compatible with all and only the possible valuations of its atomic propositions which are compatible with the truth of P in every possible circumstance. So the type of truth conditions of complete propositions is $\#((a\#t)t)$ in my logic of sense. $U_{\#(a\#t)t} = (Circumstances \to P(U_a \to P(Circumstances))$.

As Wittgenstein pointed out in the *Tractatus*, there are two limit cases of truth conditions. Sometimes the truth of a proposition is compatible with all possible ways in which objects could be. It is a tautology. Sometimes it is incompatible with all of them. It is a contradiction. In my approach, a ***tautology*** is a proposition whose truth in any circumstance is compatible with all the possible truth conditions of its atomic propositions. And a ***contradiction*** a proposition whose truth is compatible with none. By virtue of their logical form, tautologies are then true according to all possible valuations of atomic propositions and contradictions according to none. Thus tautologies are a particular case of necessarily true propositions just as contradictions are a particular case of necessarily false propositions. Unlike what is the case for other necessarily true or necessarily false propositions, we *a priori* know that tautologies are necessarily true and that contradictions are necessarily false when we apprehend their logical form. Tautologyhood and contradiction are epistemic as well as logical and metaphysical notions.

2.4. The new criterion of propositional identity

Identical propositions have the same structure of constituents and their truth in each circumstance is compatible with the same possible truth conditions of their atomic propositions. The type p of propositions is (at)(#((a#t)t)t). Thus the set Up of propositions is included in the set $PU_a \times (Circumstances \rightarrow P(U_a \rightarrow PCircumstances)$. From a logical point of view, each proposition P has a characteristic finite set of atomic propositions (in symbols id1P) and a characteristic intension (in symbols $\mathrm{id}_2\mathrm{P}$) which associates with any possible circumstance the set of possible valuations of its atomic propositions which are compatible with its truth in that very circumstance. My criterion of propositional identity is stronger than that of modal, temporal, intensional and relevance logics. Strictly equivalent propositions composed out of different atomic propositions are no longer identified. We do not make the same predications in expressing them. So the propositions that the morning star is the morning star and that the evening star is the evening star are different tautologies. Their propositional constituents are different.

Furthermore, unlike Parry[24] I do not identify all strictly equivalent propositions with the same structure of constituents. Consider the elementary proposition that the biggest whale is a fish and the contradiction that the biggest whale is and is not a fish. They are both necessarily false and contain the same single atomic proposition. But they do not have the same cognitive value. We can believe that whales are fishes. But we could not believe that whales are and are not fishes. In my logic, such propositions are different because their truth is not compatible with the same possible truth conditions of their atomic proposition[25]. However, as we will see later, my criterion of propositional identity is less rigid than that of intensional isomorphism in Cresswell's hyperintensional logic[26]. For all Boolean laws of idempotence, commutativity, distributivity and associativity of truth functions remain valid laws of propositional identity.

2.5. The set of propositions is recursive

Elementary propositions are the simplest propositions. They contain a single atomic proposition and are true in all circumstances where that atomic proposition is true[27]. *All other propositions are more complex: they are obtained by applying to simpler propositions operations which change their structure of constituents or truth conditions.* Truth functions are the simplest propositional operations. Complex propositions composed by truth functions have all and only the atomic propositions of their arguments. And their truth value in a circumstance only depends on the truth values of their arguments in that very circumstance. Thus the conjunction $P \wedge Q$ and the disjunction $P \vee Q$ of two propositions have the atomic propositions of their arguments P and Q. They only differ by their truth conditions[28].

Unlike truth functions, quantification, modal and temporal operations on propositions change the structure of constituents as well as truth conditions. When we think that all objects are such that God has knowledge of them, we predicate of Him the property of omniscience, namely that He knows everything. Such a generalized proposition contains a new atomic proposition predicating a generalization of the attribute of its argument. It is moreover true in a circumstance when that new atomic proposition is true in that circumstance[29].

Modal and temporal operations also change the structure of constituents. They add new atomic propositions. When we think that it is universally necessary that God does not make mistakes, we do more than attribute to God the property of not making mistakes. We also attribute to Him the modal property of infallibility, namely that He does not make mistakes in any possible circumstance. The property of infallibility is the necessitation of the property of not making mistakes. Modal propositions according to which it is necessary that things are such and such contain new atomic propositions predicating the necessitation of attributes of their argument. Unlike quantification, modal and temporal operations are however intensional: the truth value of a modal or temporal proposition in a circumstance depends on the truth values of its argument in other possible circumstances. Thus the truth in a circumstance of the temporal proposition that it has always been the case that P is compatible with all possible valuations of its atomic propositions which are compatible with the truth of P in all anterior circumstances.

3. TRUTH ACCORDING TO PREDICATION

Thanks to the new explication of the logical type of proposition, my logic of sense and denotation offers **a new concise definition of truth by correspondence** and articulates better the logical structure of propositions. In the philosophical tradition, from Aristotle to Tarski, truth is based on correspondence with reality: true propositions correspond to existing facts. Objects of reference have properties and stand in relations in actual and possible circumstances. Atomic propositions have therefore a well determined truth value in each circumstance depending on the actual denotation of their attributes and concepts and the order of predication.

Moments of time represent *possible complete states of the actual world* at an instant in the logic of ramified time and action. The past is unique but the future is open. We, human agents, live in an indeterminist world. So various alternative incompatible moments could directly follow a moment[30]. *Actual moments of time some represent actual complete states of the world.* The *present moment*, which represents "all nature now" (as Whitehead says), is actual as well as all moments which are anterior to it. These moments belong to the *actual course of history of this world.* Atomic propositions which are true at a moment (or time interval) in the actual course of history of the world represent facts (states of affairs, events or actions) which exist or happen at some instant (or time interval) in *the actual world.* As Wittgenstein noticed at the beginning of the ***Tractatus***, "The world divides into facts" (1.2) not into objects. Because of indeterminism, there are a lot of possible historic continuations of the present moment. We ignore which of them will turn to be actual. We can just assume that there is a single one (if the world continues).

Many possible circumstances that we consider belong to a possible non actual course of history of the world. Such non actual circumstances are however *possible.* They belong to what I will call the *logical space of reality.* Propositions which are true in possible circumstances represent possible facts that would exist if the possible course of history to which they belong were actual. **The world is part of reality.** So in order to describe the real world and represent existing facts (for example, that certain objects are soluble) in actual circumstances, we need to consider facts happening in other circumstances which are just possible. A material object of this world is soluble now if and only if it would dissolve if it were put in water.

However things could have many other properties and stand in many other relations according to us in possible circumstances (whether actual or not). In addition to the ways in which things are in reality, there are the possible ways in which we can think that they could be. We do not know how the world has been until now and how it will continue. We even ignore most of what is the case now at the present moment. So, as I explained above, we consider a lot of possible truth conditions of atomic propositions different from their actual truth conditions in thinking and speaking. In our mind, the truth of propositions is compatible with many possible ways in which we can represent objects.

However, in order that a proposition be true in a possible circumstance, things must be in that circumstance as that proposition represents them. Otherwise, there would be no correspondence with reality. Along these lines, **I propose to define as follows the concept of truth**: *a proposition is true in a circumstance when its truth in that circumstance is compatible with valuations that assign actual truth conditions to all its atomic propositions*. For short, a proposition P is true in a circumstance m/h when a possible valuation $val+ \in id_2P(m/h)$ is such that $val + (u_a) = id_2(u_a)$ for all atomic propositions $u_a \in id_1P$. As one can expect, the truth of the proposition P in that circumstance is then compatible with all the real valuations of its propositional constituents. In particular any true proposition P is true according to the real valuation val^*+ of atomic propositions. Classical laws of truth theory follow from this concise definition[31].

3.1. Cognitive aspects in the theory of truth

A speaker a often rightly or wrongly believes at a moment m that certain propositional constituents could only have such and such possible denotations in circumstances. In that case, atomic propositions composed from such constituents could only be true according to him Formal Ontology, Propositional Identity and Truth 17 or her at that moment in such and such sets of possible circumstances. A particular set $Val(a, m)$ of possible valuations of atomic propositions is then compatible with what the speaker a believes at the moment m. Any speaker having in mind atomic propositions believes in the truth of certain propositions containing them. **One can define exactly the notion of truth according to a speaker** in my logic of sense: A *proposition is true in a circumstance m/h according to a speaker a at a moment m*

when the truth of that proposition in that circumstance is compatible with all possible valuations assigned by that agent at that moment to its propositional constituents i. e. when $Val(a,m) \subseteq id_2P(m/h)$.

By hypothesis, tautological propositions are true and contradictory propositions are false according to all agents who have them in mind. But impossible propositions which are not contradictory can be true and necessarily true propositions which are not tautological can be false according to agents at some moments. These are basic principles of my epistemic logic. **So the logic of language impose different limits to reality and thought.** On one hand, all propositions which are false in all possible circumstances represent *impossible facts* that could not exist in reality. (I do not advocate the need in formal semantics of impossible circumstances where such impossible facts could exist.) On the other hand, I admit that there are necessarily false propositions that we can believe (e. g. that whales are fishes). So we can represent impossible facts and wrongly think that they exist. Among necessarily false propositions, I distinguish then those that we can believe from others that we cannot (the pure contradictions). All depends whether their truth is compatible or not with some valuations of their atomic propositions.

The notion of strong implication

Human beings are not perfectly rational. We are often inconsistent. Furthermore, we do not make all valid inferences. We assert (and believe) propositions without asserting (and believing) all their logical consequences. Thus, our illocutionary (and psychological) commitments are not as strong as they should be from the logical point of view. However, we, human agents, are not totally irrational. On the contrary, **we manifest *a minimal rationality*[32] in thinking and speaking.** When we know by virtue of competence that a proposition is false we never relate it to the world with the intention of achieving a success of fit between words and things. So we do not attempt to perform unsatisfiable illocutionary acts with a non empty direction of fit and a contradictory propositional content. Such acts are imperformable because we a priori know that they cannot be satisfied.

Moreover, when we a priori know by virtue of competence that a proposition cannot be true unless another is also true, we cannot assert (or believe) that proposition without asserting (or believing) the second. There is in philosophical logic an important *relation of strict* *implication* between propositions that is due to C. I. Lewis. By definition, a proposition *strictly implies* another proposition whenever that other proposition is true in all possible circumstances where it is true. Hintikka[33] and others have advocated that belief and knowledge are closed under strict implication. However, from a cognitive point of view, we ignore how propositions are related by strict implication, just as we ignore in which possible circumstances they are true. Any proposition strictly implies infinitely many necessarily true propositions. However we could not assert (or believe) all of them in any circumstance.

We need a relation of implication much finer than strict implication in order to explicate existing illocutionary and psychological commitments. Thanks to the predicative analysis of the logical form of propositions, **one can define a relation of implication called strong implication that is finer than all others.** *A proposition strongly implies another proposition when* firstly, it contains all its atomic propositions and secondly, all possible valuations of atomic propositions which are compatible with its truth in a circumstance are also compatible with the truth of that other proposition in that very circumstance. In other words, P *strongly implies* Q (in symbols $P \mapsto Q$) when $id_1P \supseteq id_2Q$ and, for any circumstance m/h, $id_1P(m/h) \subseteq id_2Q(m/h)$.

Unlike strict implication, *strong implication is cognitive.* Whenever a proposition P strongly implies another proposition Q we cannot apprehend that proposition P without knowing *a priori* that it strictly implies the other Q. For in apprehending P, we have by hypothesis in mind all atomic propositions of Q. We make all the corresponding acts of reference and predication. Furthermore, in understanding the truth conditions of proposition P, we consider all possible valuations of these atomic propositions which are compatible with its truth in any circumstance m/h. The same possible valuations of atomic propositions of Q which are in P are then by hypothesis compatible with the truth of proposition Q in that circumstance m/h. Thus, in expressing P, we know that Q follows from P when P strongly implies Q. According to my epistemic logic, belief and knowledge are then closed under strong rather than strict implication.

As I will show, *strong implication obeys a series of important universal laws.* Unlike strict implication, strong implication is anti-symmetrical. Two propositions which strongly imply each other are identical. Unlike Parry's analytic implication, strong implication is always tautological. In my terminology, a proposition P *tautologically implies* another Q when, for any circumstance m/h, $id_1 P(m/h) \subseteq id_2 Q(m/h)$. Natural deduction rules of elimination and introduction generate strong implication when and only when all atomic propositions of the conclusion belong to the premises. So a proposition P does not strongly imply any disjunction of the form $P \vee Q$. Moreover strong implication is *paraconsistent*. A contradiction does not strongly imply all propositions. Finally, strong implication is *finite* and *decidable*. (More on this later.)

4. ANALYSIS OF MODAL AND TEMPORAL PROPOSITIONS

Let us now analyze in terms of predication the logical form of modal and temporal propositions in the framework of the logic of ramified time. We need to take into account the following facts:

1. *As regards their structure of constituents*

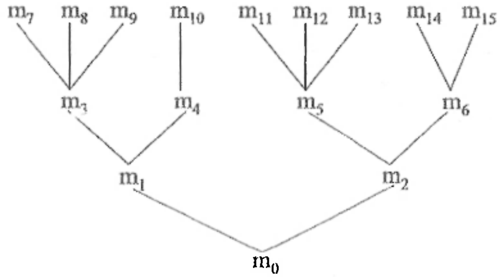
Unlike truth functions, modal and temporal operations on propositions enrich the set of their atomic propositions. As I said earlier, we predicate modal and temporal attributes in expressing modal and temporal propositions. For example, in asserting that Paul was previously married to Julie we predicate of Paul the temporal property of being an ex-husband of Julie.

2. *As regards their truth conditions*

As Occam already pointed out, in order to analyze the truth conditions of future propositions we need to consider not only moments of time but also histories. In the logic of branching time, a *moment* is a possible complete state of the world at a certain instant and the *temporal relation of anteriority / posteriority* between moments is partial rather than linear because of indeterminism. On the one hand, there is a single causal route to the past: each moment m is preceded by at most one past moment m'. And all moments are historically connected: any two distinct moments have a common historical ancestor in their past. On the other hand, there are multiple future routes: several incompatible moments might follow upon a given moment. For facts existing at a moment can have incompatible future effects.

Consequently, the set of moments of time in any interpretation has the formal structure of a *tree-like frame* which can be represented as follows:

A maximal chain h of moments of time is called a *history*. It represents a possible course of history of the actual world. There are nine different histories in the preceding figure. A first history h_1 goes from moment m_0 to moment m_7, a second one h_2 from moment m_0 to moment m_8, and so on. The truth of certain propositions is settled at each moment no matter which historical continuation of that moment is under consideration. So are past propositions because all histories through



a moment have the same past at that moment. The past proposition that it was the case that A (in symbols: $WasA$) is true at a moment m according to any history when A is true at a moment m' anterior to m. Thanks to histories, branching temporal logic can analyze the notion of settled truth. The proposition that it is settled that A (in symbols Settled$A$) is true at a moment m according to a history h if and only if the proposition that A is true at that moment m according to all histories to which it belongs. Unlike what is the case for past propositions, the truth of future propositions is not settled at each moment; it depends on which historical continuation h of that moment is under consideration. Following Peirce and Belnap [1994] I will say that the future proposition that it will be the case that A (in symbols Will$A$) is true at a moment m according to a history h when the proposition that A is true at a moment m' posterior to m according to that very history h[34].

Given the causal ordering relation, some histories h and h' are *undivided* at certain moments m; they have the same present and past at these moments. In that case, moment m and all moments m' anterior to that moment belong to both histories h and h'. The relation of having the same present and past at a moment m is an equivalence relation which partitions the set of histories to which m belongs into a family of exhaustive and pairwise disjoint subsets, each of which keeps undivided histories at the next moment together. Each set of histories in the partition is an *elementary immediate possibility* after m. If there is only one such subset in the partition, the moment m is *deterministic*. Otherwise, it is *undeterministic*.

Two moments of time are *alternative* in my terminology when they belong to histories which have the same past before these moments. For example, moments m_7, m_8 and m_9 are alternative in the last figure. They represent how the world could be immediately after the moment m_3[35]. The set Instant of all instants is a partition of the set *Time* of all moments of time which contains exactly one moment of each history and respects the temporal order of histories. Moments which belong to the same instant are said to be *coinstantaneous*. So the first instant is the singleton containing the moment of time which is anterior to all others (such a first moment exists in an interpretation if the world has a beginning according to it). And after each instant ι the next instant ι' is the set that contains all and only the alternative moments that follow the moments of that instant ι. For example, moments m_3, m_4, m_5 and m_6 of figure 1 are coinstantaneous.

Thanks to instants, branching logic can analyze important modal notions such as *historic necessity* (in the sense of now unpreventability)[36] and *historic possibility*. Consider the proposition that it is then necessary that A (in symbols $\Box A$) in the sense that it could not have been otherwise than A: it is true at a moment m according a history h when the proposition that A is true at all moments m' coinstantaneous with m according to the histories h' to which they belong. Whenever $\Box A$ is true at a moment m, A represents a fact that is not only settled but also *inevitable* at that moment. Similarly, the proposition that it is then possible that A (in symbols $\Diamond A$) is true at a moment m according a history h when the proposition that A is true at some moment m' coinstantaneous with m according to at least one history h'.

The ideal object language

The ideal object propositional language L of my logic of ramified time and historic modalities contains the following syntactic resources:

Vocabulary of L

Language L contains in its lexicon:

(1) a series of propositional symbols

$p, p', p'', \ldots, q, q', q''', \ldots$ and (3) the syncategorematic expressions: *Tautological* , $>$, $\wedge$, $\Box$, *Will*, *Was*, $\neg$, (and).

Rules of formation of L

The set L_p of propositional formulas.

Propositional symbols are propositional formulas. If A_p and B_p are propositional formulas then $\neg A_p$, $\Box A_p$, $Will A_p$, $Was A_p$, $Tautological(A_p)$, $(A_p > B_p)$ and $(A_p \wedge B_p)$ are new complex propositional formulas.

Propositional symbols express propositions.

A formula of the form $\neg A_p$ expresses the negation of the proposition expressed by A_p.

$\Box A_p$ expresses the modal proposition that A_p is then necessary (i. e. that it could not have been otherwise than A_p).

$Will A_p$ expresses the future proposition that it will be the case that A_p.

$Was A_p$ expresses the past proposition that it has been the case that A_p. $Tautological(A_p)$ expresses the proposition that A_p is tautological.

$(A_p > B_p)$ expresses the proposition that all atomic propositions of B_p are atomic propositions of A_p.

$(A_p \wedge B_p)$ expresses the conjunction of the two propositions expressed by A_p and B_p.

Rules of abbreviation

Exterior parentheses will often be omitted. Parentheses will also be omitted according to the rule of the association to the left. Truth, modal and temporal connectives are introduced according to usual rules.

Disjunction: $(A_p \vee B_p) =_{df} \neg(\neg Ap \wedge \neg Bp)$ *whenever* A *and* $B \in L_p$

Material implication: $(A_p \Rightarrow B_p) =_{d} f \neg(A_p \wedge \neg B_p)$

Material equivalence: $(A_p \Leftrightarrow B_p) =_{df} (A_p \Rightarrow B_p) \wedge (B_p \Rightarrow A_p)$

$Was\text{-}always A_p =_{df} \neg Was \neg A_p$

$Will\text{-}always A_p =_{df} \neg Will \neg A_p$

$Always A_p =_{df} Was\text{-}always A_p \wedge A_p \wedge Will\text{-}always A_p$

$Sometimes A_p = Was A_p \vee A_p \vee Will A_p$

Historical possibility: $\Diamond A_p =_{df} \neg\Box\neg A_p$

Universal necessity: $\blacksquare A_p =_{df} Always \Box A_p$

Universal possibility: $\blacklozenge A_p =_{df} \neg\blacksquare\neg A_p$

Strict impliccation: $A_p -\in B_p =_{df} \Box(A_p \Rightarrow B_p)$

Here are new abbreviations:

Analytic implication[37]*:* $A_p \rightarrow B_p =_{df} (A_p > B_p) \wedge (A_p -\in B_p)$

Strong implication: $A_p \mapsto B_p =_{df} (A_p > B_p) \wedge Tautological(A_p \Rightarrow B_p)$

Same structure of constituents: $A_p \equiv B_p =_{df} (A_p > B_p) \wedge (B_p > A_p)$

Propositional identity: $A_p = B_p =_{df} (A_p \mapsto B_p) \wedge (B_p \mapsto A_p)$

5. THE FORMAL SEMANTICS

One must interpret formulas of L and associate with them truth conditions according to the following rules. A standard model for L is a quintuple M of the form $< Time, Instant, Atom, Val, \|\ \| >$, where

(1) *Time* is a non empty set whose elements $m, m', m'' \ldots$ are moments which represent possible complete states of the world. $\leq$ is a partial order on the set *Time* representing the *causal ordering relation* or the *temporal relation of anteriority / posteriority*. $m < m'$ means that moment m is in the past of moment m' and that moment m' is in the future of possibilities of m. By definition, $<$ is subject to *historical connection* and *no downward branching*. Any two distinct moments m and m' have a common historical ancestor: some moment m''' such that $m'' < m$ and $m'' < m'$. Moreover, the past is unique: if there is a moment m'' such that $m < m''$ and $m' < m''$ then either $m = m'$ or $m < m'$ or $m' < m$.

Consequently, $(Time, =)$ is a tree-like frame. A maximal chain h of moments of *Time* is called a *history*. It represents a possible course of history of the world. Let *History* be the set of all histories. Two histories h and h' are *undivided* at a moment m when they have the same present and past at these moments, that is to say when, for all moments $m' \leq m$, $m' \in h$ and $m' \in h'$.

(2) The set *Instants*, whose elements $\iota, \iota', \ldots$ are called *instants*, is a partition of the set *Time* which satisfies *unique intersection* and *order preservation*. So for all ι and h there is a unique moment m (in symbols $m(\iota, h)$ belonging to ι and h. And $m(\iota, h) \leq m(\iota', h)$ when $m(\iota, h') \leq m(\iota', h')$. Two moments of time m and m' are coinstantaneous (in symbols: $m \doteq m'$) when they belong to the same instant. Any pair of coinstantaneous moments m and m' represent two complete possible states of the world in which things could be at a certain instant.

(3) *Atoms* is an infinite set whose elements are *atomic propositions*. In the present logic, atomic propositions are left undefined. However, as I have explained earlier, one can define their formal nature using the modal theory of types.

$P[Atoms]$ is an upper modal temporal and agentive semi lattice containing finite sets of atomic propositions which is closed under union $\cup$ and a unary modal and temporal operation * satisfying the following conditions: For any $\Gamma_a \subseteq Atoms$, $\Gamma_a \subseteq^* (\Gamma_a)$ and for any Γ_1 and $\Gamma_2 \subseteq Atoms$, $^*(\Gamma_1 \cup \Gamma_2) =^* (\Gamma_1) \cup^* (\Gamma_2)$ and $^{**}(\Gamma_1) =^* (\Gamma_1)$[38].

All the elements of $P(Atoms)$ are finite sets of atomic propositions from which expressible propositions can be composed.

(4) Val is the set of all functions from $Atoms$ into $P(Time \times History)$. Its elements are *valuations of atomic propositions.* Each valuation $val \in Val$ assign possible truth conditions to atomic propositions. $m, h \in val(u_a)$ means that atomic proposition u_a is true at moment m according to history h under that valuation. When $m, h \in val(u_a), m \in h$. I will often indicate this by writing m/h. There is a distinguished valuation $val_M \in Val$ determining the actual truth conditions that atomic propositions have under the model M. So, for any atomic proposition u_a, $val_M(u_a)$ is the set of all pairs m/h of moments and histories where that atomic proposition is true according to the model M.

(5) The set U_p of all *propositions* which are expressible in L according to M is an infinite subset of the Cartesian product $P[Atoms| \times P$ $(Val)^{Time \times History}$. The first term, $[P|$, of a proposition P represents the *set of its atomic propositions.* And its second term, $|P|$, its *truth conditions.* So for each moment m and history h, where $m \in h, |P|_{m/h}$ is the set containing all *possible valuations of atomic propositions which are compatible with the truth of* proposition P at the moment m according to the history h.

(6) $||\ |$ is an interpreting function which associates with each propositional formula the proposition that that formula expresses according to model M. The proposition $||A_p||$ expressed by formula $A_p \in L_p$ in the model M is a pair $< [A_p], |A_p| >$ belonging to the Cartesian product $P[Atoms] \times P(Val)^{Time \times History}$.

Proposition $||A_p||$is defined inductively as follows:

(i) For any propositional symbol p, $||p|| \in U_p$.

(ii) $[Tautological\ B_p] = [B_p]$ and $|Tautological\ B_p|_{m/h} = Val$ when $|B_p|_{m/h} = Val. Otherwise, |Tautological\ B_p|_{m/h} = \emptyset$.

(iii) $[\neg B_p] = [B_p]$ and $|\neg B_p|_{m/h} = Val - |B_p|_{m/h}$.

(iv) $[\Box B_p] = {}^*[B_p]$ and $|\Box B_p|_{m/h} = \bigcap_{m'/h'} \{|B_p|_{m'/h'} \text{ where } m' \cong m\}$.

(v) $[Will B_p| =^* [B_p]$ and $|Will A_p[_{m/h} = \bigcup_{m'>m} |A_p|_{m'/h}$.

(vi) $[Was B_p] =^* [B_p]$ and $|Was Ap|_{m/h} = \bigcup_{m'<m} |A_p|_{m'/h}$.

(vii) $[B_p \wedge C_p] = [B_p] \cup [C_p]$ and $|B_p \wedge C_p|_{m/h} = |B_p|_{m/h} \cap |C_p|_{m/h}$.

(viii) $[B_p \geq C_p] = [B_p] \cup [C_p]$ and $|B_p \geq Cp|_{m/h} = Val$ when $[B_p] \supseteq [C_p]$.

Otherwise $|B_p \geq C_p|_{m/h} = \emptyset$.[39]

Definition of truth and validity

A proposition $||A_p||$ is *true at a moment* m according to a *history* h under the model M when the real valuation $val_M \in |A_p|_{m/h}$. A formula A_p of L is *valid* or *logically true* (in symbols: $\models A_p$) when $||A_p||$ is true at all moments according to all histories in all standard models M of L.

6. A COMPLETE AXIOMATIC SYSTEM

I conjecture that all and only the valid formulas of my logic are provable in the following axiomatic system S.

The axioms of S are all the instances in L of the following axiom schemas :

Classical truth functional logic

(T1) $(A_p \Rightarrow (B_p \Rightarrow A_p))$

(T2) $((A_p \Rightarrow (B_p \Rightarrow C_p)) \Rightarrow ((A_p \Rightarrow B_p) \Rightarrow (A_p \Rightarrow C_p)))$

(T3) $((\neg A_p \Rightarrow \neg B_p) \Rightarrow (B_p \Rightarrow A_p))$

S5 Modal logic

(T4) $(\Box A_p \Rightarrow A_p)$

(T5) $(\Box(A_p \Rightarrow B_p) \Rightarrow \Box(A_p \Rightarrow \Box B_p))$

(T6) $(\Diamond A_p \Rightarrow \Box\Diamond A_p)$

Axioms for tautologies

(T7) $(Tautological\ (A_p)) \Rightarrow \Box A_p$

(T8) $(Tautological\ (A_p)) \Rightarrow (A_p = (A_p \Rightarrow A_p))$

(T9) $(\neg Tautological\ (A_p)) \Rightarrow ((Tautological\ (A_p)) = (A_p \wedge \neg A_p))$

(T10) $Tautological\ (A_p) \Rightarrow (Tautological\ (A_p \Rightarrow B_p))$

(T11) $Tautological\ (A_p) \Rightarrow Tautological\ (\Box A_p)$

(T12–13) $Tautological\ (A_p) \Rightarrow Tautological\ (\textit{Will-always}\ A_p)$ And similarly for *Was-always*

(T14) $(A_p = B_p) \Rightarrow Tautological\ (C \Rightarrow C^*)$ where C^* and C are propositional formulas which differ at most by the fact that an occurrence of B_p replaces an occurrence of A_p.

Axioms for propositional composition

(C1) $(A_p > B_p) \Rightarrow Tautological\ (A_p > B_p)$

(C2) $\neg(A_p > B_p) \Rightarrow Tautological \neg(A_p > B_p)$

(C3) $A_p > A_p$

(C4) $(A_p > B_p) \Rightarrow ((B_p > C_p) \Rightarrow (A_p > C_p))$

(C5) $(A_p \wedge B_p) > A_p$
(C6) $(A_p \wedge B_p) > B_p$
(C7) $(Cp > A_p) \Rightarrow ((C_p > B_p) \Rightarrow (C_p > (A_p \wedge B_p)))$
(C8) $A_p \equiv \neg A_p \wedge (A_p = Tautological\ A_p) \wedge ((A_p \wedge B_p) \equiv (A_p > B_p))$
(C9) $\Box A_p > A_p$
(C10) $\Box\neg A_p \equiv \Box A_p$
(C11) $\Box(A_p \wedge B_p) \equiv (\Box A_p \wedge \Box B_p)$
(C12) $\Box\Box A_p \equiv \Box A_p$
(C13) $Will A_p \equiv \Box A_p$
(C14) $Was A_p \equiv \Box A_p$

Branching time logic

(TL1) (*Will-always*$(A_p \Rightarrow B_p) \Rightarrow$ (*Will-always* $A_p \Rightarrow$ *Will-always*B_p))

(TL2) (*Was-always* $(A_p \Rightarrow B_p) \Rightarrow$ (*Was-always* $A_p \Rightarrow$ *Was-always* B_p))

(TL3) ($A_p \Rightarrow$ *Was-always* $Will A_p$)
(TL4) ($A_p \Rightarrow$ *Will-always* $Was A_p$)
(TL7) ($Was A_p \Rightarrow$ *Will-always* $Was A_p$)
(TL8) $Will A_p \Rightarrow$ *Will-always* $(Will A_p \vee A_p \vee Was A_p)$
(TL9) $Was A_p \Rightarrow$ *Was-always* $(Will A_p \vee A_p \vee Was A_p)$

Historic modality with time

(MT1) ($Was\Box(A_p \wedge$ *Will-always* $B_p) \wedge$ *Was-always* $\neg(B_p \wedge \Diamond C_p)) \Rightarrow \Box($*Will-always* $D_p \wedge Was C_p \Rightarrow Was(A_p \wedge (C_p \vee Was C_p) \wedge$ *Will-always* $(C_p \Rightarrow$ *Will-always* $D_p)))$.

(MT2) (*Was-always* $(A_p \wedge$ *Was-always*$\neg(B_p \wedge \Diamond C_p) \wedge Will(B_p \wedge A_p \wedge \Diamond D_p)) \wedge Was(\Box E_p \wedge$ *Will-always* $B_p)) \Rightarrow \Box($*Will-always* $Q_p \Rightarrow Was(E_p \wedge$ *Will-always* $(C_p \Rightarrow$ *Will-always* $D_p \Rightarrow$ *Will-always* $Q_p)))$[40].

The rules of inference of axiomatic system S are:

The rule of Modus Ponens: From the sentences $(A \Rightarrow B)$ and A infer B. The *necessitation rules*: From a theorem A infer *Tautological* A.

7. VALID LAWS

7.1. Laws of composition

A proposition is composed from all the atomic propositions of its constituent propositions.

Thus $| = A_p > p$ when p occurs in A_p.

There is a law of distribution of the constituent atomic propositions of modal and temporal propositions with respect to truth functions. $| = M(A_p \wedge B_p) = (MA_p \wedge MB_p)$ where M is of the form $\Box$, $\neg\Box$, $\Box\neg$, $\neg\Box\neg$, $Will\neg$, $Was, Was\neg$, $\neg Will$ or $\neg Was$. So all the different modal and temporal propositions of the form MA_p have the same atomic propositions.

$| = MA_p \equiv M'A_p$, where M and M' are $\Box$, $\neg\Box$, $\Box\neg$, $\neg\Box\neg$, $Will\neg$, $Was, Was\neg$, $\neg Will$ or $\neg Was$.

7.2. Laws for tautologyhood

There are modal, temporal as well as truth functional tautologies. Thus $| = Tautological\ (\Box A_p \Rightarrow A_p)$ and $| = Tautological\ (A_p \Rightarrow Will - always\ WasA_p)$.

Tautologyhood is stronger than logical necessity.

Thus $| = Tautological\ (A_p) \Rightarrow \blacksquare A_p$.

But $| \neq \blacksquare A_p \Rightarrow Tautological\ (A_p)$. Necessarily true elementary propositions like the proposition that whales are mammals are not tautological.

7.3. Laws for tautological implication

Tautological implication is finer than *strict implication.*

$| = Tautological\ (A_p) \Rightarrow B_p) \Rightarrow (A_p^- \in B_p)$. But the converse is not true. For example, the elementary proposition that the biggest whale is a fish strictly implies the contradiction that it is and that it is not a fish. But it does not tautologically imply that contradiction. Only elementary propositions whose atomic proposition is it self contradictory tautologically imply a contradiction. Whenever a proposition tautologically implies another, we can express the first without expressing the second. (Suppose the second contains new atomic propositions.) But one cannot have both in mind without knowing that the first imply the second.

7.4. Laws for strong implication

Strong implication is the strongest kind of propositional implication. By definition, $A_p \mapsto B_p = (A_p > B_p) \wedge Tautological\ (A_p \Rightarrow B_p)$

Whenever a proposition strongly implies another proposition, this is tautological. $| = (A_p \mapsto B_p) \Leftrightarrow Tautological\ (A_p \mapsto B_p)$.

Any proposition implying strongly another proposition is identical with its conjunction with that other proposition. So $| = (A_p \mapsto B_p) \Leftrightarrow ((A_p \wedge B_p) = A_p)$.

There are two causes of failure of strong implication:

Firstly, $| = \neg(A_p > B_p) \Rightarrow \neg(A_p \mapsto B_p)$. A proposition does not strongly imply any proposition composed from other atomic propositions: In that case, it is possible to have in mind the first proposition without having in mind the second.

Secondly, $| = \neg Tautological\ (A_p \Rightarrow B_p)\neg(A_p \mapsto B_p)$. A proposition does not strongly imply any proposition that it does not tautologically imply. In that case, we do not necessarily know by virtue of linguistic competence that the first proposition has more truth conditions than the second.

Unlike strict implication, strong implication is a relation of partial order. So Parry's *analytic implication*, which is not anti-symmetric, is weaker than strong implication. $\models (A_p \mapsto B_p) \Rightarrow (A_p \rightarrow B_p)$. But $\not\models (A \rightarrow B) \Rightarrow (A \mapsto B)$. For $\not\models (A \rightarrow B) \Rightarrow Tautological\ (A \Rightarrow B)$.

Paradoxical laws of the following kind do not hold for strong implication:

$\models ((\Box p \vee \neg\Box p) \rightarrow \Box p) \vee (\Box p \rightarrow (\Box p \wedge \neg\Box p))$.

7.5. Natural deduction

All and only the valid laws of inference of modal and temporal logic where the premises contain all atomic propositions of the conclusion are valid laws of strong implication. This leads to the following *system of natural deduction* for strong implication:

The law of elimination of conjunction:

$\models (A_p \wedge B_p) \mapsto A_p$ and $\models (A_p \wedge B_p) \mapsto B_p$.

The law of *elimination of disjunction:*

$\models ((A_p \mapsto C_p) \wedge (B_p \mapsto C_p)) \Rightarrow (A_p \vee B_p) \mapsto C_p$.

Failure of the law of introduction of disjunction: $| \neq A_p \mapsto (A_p \vee B_p)$.

So strong implication is stronger than entailment in the sense of the logic of relevance. For the law of introduction of disjunction holds for entailment.

Failure of the law of elimination of negation: $\mid\neq (A_p \wedge \neg A_p) \mapsto B_p$.

Strong implication is *paraconsistent.*

The law of *elimination of material implication:* $\models (A_p \wedge (A_p \Rightarrow B_p)) \mapsto B_p$.

The law of *elimination of necessity:* $\models \Box A_p \mapsto A_p$.

The law of *elimination of always:* $\models$ *Always* $A_p \mapsto A_p$.

The law of *introduction of necessity:* $\models (A_p \mapsto B_p) \Rightarrow (\Box A_p \mapsto \Box B_p)$.

And similarly for *Will, Was.*

Failure of the law of elimination of possibility: $\mid\neq (\Diamond A_p \mapsto B_p) \Rightarrow A_p \mapsto B_p$.

Failure of the law of introduction of possibility: $\mid\neq A_p \mapsto \Diamond A_p$.

For $\mid\neq A_p > \Diamond A_p$. And similarly for *Sometimes.*

Strong implication is decidable. For $\models A_p > B_p$ when all propositional symbols which occur in B also occur in A. Moreover, $\models$ *Tautological* $(A_p \Rightarrow B_p)$ when semantic tableaux for $(A_p \Rightarrow B_p)$ close[41].

There is also *a theorem of finiteness for strong implication:*

Every proposition only strongly implies a finite number of other propositions. In particular, a proposition strongly implies all and only the tautologies which are composed from its atomic propositions.

$\models$ *Tautological* $B_p \Rightarrow (A_p \mapsto B_p \Leftrightarrow A_p > B_p)$.

Similarly, a contradiction strongly implies all and only the propositions composed from its atomic propositions. $\models$ *Tautological*$\neg A_p \Rightarrow (A_p \mapsto B_p \Leftrightarrow (A_p > B_p))$. The fact that knowledge is closed under strong implication is confirmed by the decidability and finiteness of strong implication.

7.6. Laws of propositional identity

Modal and temporal propositions are composed from several atomic propositions. So $\mid\neq (\Box A_p = p)$ for any propositional symbol p. And similarly for *Will* and *Was.*

The failure of this law is shown in language use. Properties such as being identical with itself are possessed by all objects in all circumstances. They have the same extension as their necessitation. But when we think that Oedipus is identical with himself, we do not eo ipso think that it is necessary that he be identical with himself. So modal propositions are not reducible to elementary propositions.

All the classical *Boolean laws of idempotence, commutativity and associativity* remain valid:

$\models A_p = (A_p \wedge A_p)$;

$\models (A_p \wedge B_p) = (B_p \wedge A_p)$ and $\models (A_p \wedge (B_p \wedge C_p)) = ((A_p \wedge B_p) \wedge C_p)$.

As well as the *laws of distributivity:*

$\models \neg(A_p \vee B_p) = (\neg A_p \wedge \neg B_p)$;

$\models A_p \wedge (B_p \vee C_p) = (A_p \wedge B_p) \vee (A_p \wedge C_p)$;

$\models \Box(A_p \wedge B_p) = (\Box A_p \wedge \Box B_p)$.

And *the laws of reduction:*

$\models \neg\neg A_p = A_p$ and $\models M\Box A_p = \Box A_p$ and $\models M\Diamond A_p = \Diamond A_p$ where $M = \Box, \Box\neg, \Diamond$ or $\Diamond\neg$.

In particular, $\models \Box A_p = \Box\Box A_p$.

Identical propositions need not be *intensionally isomorphic* in the sense of hyperintensional logic[42]. As I have argued in [1990–91], intensional isomorphism is a too strong criterion of propositional identity.

However, propositional identity requires more than *co-entailment* in the sense of the logic of relevance. For $\not\models A \mapsto (A \wedge (A \vee B))$. M. Dunn[43] regrets that A and $(A \wedge (A \vee B))$ co-entail each other. For most formulas of such forms are not synonymous. Co-entailment is not sufficient for synonymy because it allows for the introduction of new senses. Strong equivalence which requires the same structure of constituents is necessary for an adequate analysis of synonymy. Finally, strong equivalence is finer than Parry's *analytic equivalence.* $\not\models (\Box p) \Rightarrow (\Box p = (\Box p \vee \neg\Box p))$ and $\not\models (\neg\Box p) \Rightarrow (\neg\Box p = (\Box p \wedge \neg\Box p))$. But such paradoxical laws hold for analytic equivalence.

Notice that the law of determinism does not hold in the logic of branching time.

So $\not\models A_p \Rightarrow$ *Was-always*$\Box Will A_p$ and $\not\models \Box A_p \Rightarrow$ *Was-always*$\Box Will A_p$.

Similarly, $\not\models Will A_p \Rightarrow \Box Will A_p$. But the following new laws hold for historic modalities[44].

$\models (Was\Box A_p \Rightarrow \Box Was\Box A_p)$ and $\models (\Box$ *Will-always*$A_p \Rightarrow Will-always\Box A_p)$.

Notes

1. Following Descartes (1641), I distinguish conceptual thoughts from other types of thought like perception and imagination whose contents are presentations rather than representations of facts. See the Sixth Meditation.

2. The notion of force comes from G. Frege who used the German term "Kraft". See "Gedanke" and "Verneinung" ***Beitrage zur Philosophie der deutschen Idealismus*** Volume 1, 1918–1919 and "Gedankengefüge" *Beitrage zur Philosophie der deutschen Idealismus* Volume 3, 1923–1926.

3. The most common force markers are verbal mood and sentential type. For example, declarative sentences serve to make assertions. Interrogative sentences serve to ask questions and imperative sentences to give directives.

4. See our joint book ***Foundations of illocutionary Logic*** Cambridge University Press 1985 as well D. Vanderveken ***Meaning and Speech Acts***, Volume I: ***Principles of Language Use*** and Volume II: ***Formal Semantics of Success and Satisfaction***, Cambridge University Press, 1990–1991.

5. In order to be satisfied, an elementary illocutionary act must have a true propositional content. So the traditional correspondence theory of truth for propositions is part of the more general theory of satisfaction for illocutionary acts.

6. Non declarative sentences are then also logically related by virtue of the truth conditions of propositions which are their senses. For example, the sentences "Do it!" and "You could not do it" express illocutionary acts which are not simultaneously satisfiable.

7. For a clear presentation of the differences between the two ontologies see David Kaplan "How to Russell a Frege-Church" ***The Journal of Philosophy*** 716–729, 1971.

8. The theory of types of sense and denotation underlying my stratification comes from Alonzo Church "A Formulation of the Logic of Sense and Denotation" in P. Henle *et al* (eds.) ***Structure Method and Meaning*** Liberal Arts Press 1951.

9. The law of excluded middle holds for success and unsuccess just as it holds for truth and falsity. Either an illocutionary act is performed or it is not performed in a speech situation. Failure is a special case of unsuccess which occurs only when a speaker makes an unsuccessful attempt to perform the illocutionary act.

10. The notion of circumstance comes from D. Kaplan (1979) "On the Logic of Demonstratives" Propositions are true in possible circumstances. A possible circumstance can be a moment of time, a possible world, a pair of a moment of time and history. All depends on the logic under consideration.

11. Properties are attributes of degree 1; they are satisfied by sets of (unary sequences of) individuals under concepts.

12. See D. Lewis (1972) "General Semantics".

13. The present theory of types is cumulative. Unlike Russell I admit the type of sets whose elements are of different inferior types.

14. I will only consider first order propositions in the present chapter.

15. The term and notion of intension come from Carnap (1956) ***Meaning and Necessity***, University of Chicago Press.

16. See D. Vanderveken (1999) "Success, Satisfaction and Truth in the Logic of Speech Acts and Formal Semantics" and D. Vanderveken (2001) "Universal Grammar and Speech Act Theory".

17. This example comes from a talk by D. Kaplan at McGill University. "Hesperus" and "Phosphorus" are different proper names of Venus.

18. To predicate is not to judge; it is just to apply an attribute to arguments. So acts of predication are purely propositional; they are independent from force. One can predicate a property of an object of reference in asking a question as well as in making an assertion.

19. See Kripke (1977) "Speaker's Reference and Semantic Reference".

20. In case no single individual falls under the concept uc in the circumstance m/h according to a valuation val we could also say that that valuation is undefined for that concept in that circumstance. In that case, possible valuations of individual concepts would be partial functions.

21. Of course, when the attribute R^n is extensional, atomic proposition $R_n(u_c^1, \ldots, u_c^n)$ is true according to *val* at m/h if and only if $< val(u_c^1)m/h, \ldots, val(u_c^n)m/h > \in val(R_n)m/h$.

22. It is compatible with all the possible truth conditions of that atomic proposition according to which it is true in at least one anterior circumstance.

23. Possible truth conditions of other atomic propositions do not matter. The truth of any proposition is compatible with all their possible truth conditions. Formal Ontology, Propositional Identity and Truth 31.

24. W. T. Parry (1933) "Ein Axiomsystem fuer eine neue Art von Implikation (analytische Implikation)".

25. On the one hand, a lot of possible truth conditions are compatible with the truth in any circumstance of the elementary proposition that the biggest whale is a fish, namely all those according to which its atomic proposition is true in that circumstance. So we can believe it. On the other hand, the truth of the contradiction is not compatible with any possible truth condition of its atomic proposition. Since we know this a priori in understanding its truth condition, we cannot believe it.

26. Max Cresswell (1975) "Hyperintensional Logic".

27. Thus the set of atomic propositions of any elementary proposition P is a singleton $\{u_a\}$ and its intension $u_{\#(a\#t)_t}$ is such that, for any circumstance $c, u_{\#(a\#t)t}(m/h) = \{f \in (U_a \rightarrow PCirconstances)/m/h \in f(u_a)\}$.

28. As is well known, the truth of the disjunction in a circumstance is compatible with all the possible valuations of its atomic propositions which

are compatible with the truth of at least one of its arguments in that circumstance. But the truth of the conjunction is only compatible with the possible valuations which are compatible with the truth of both arguments.

29. As I (1997) pointed out in "Quantification and the Logic of Generalized Propositions" second order predication is not needed for the logical analysis of generalization over individual objects. One can remain in the simpler formal ontology of first order attributes. Using the classical logic of attributes one can analyze new predicated attributes as being generalizations of first order attributes. The property of omniscience is a universal generalization of the relation of knowing in the intensional logic of attributes. See G. Bealer ***Quality and Concept*** (1982) for an explanation of the operations of generalization on attributes.

30. We are free because we could do something else from what we actually do.

31. See my forthcoming book ***Propositions, Truth and Thought.***

32. The term and notion of minimal rationality comes from Cherniak (1986).

33. See Hintikka ***Knowledge and Belief*** Cornell University Press (1962).

34. According to the actualist point of view (that Occam was the first to advocate). The future proposition $WillA$ is rather true at a moment m if and only if the proposition that A is true at a moment m' posterior to m according to the particular history that represents the actual historic continuation of that moment. See Belnap (2001)'s arguments against actualism.

35. We do not know how the world will continue. But among all alternative moments that could directly follow the present moment we know that one and only one will be actual (if the world continues). So among all alternative moments that follow immediately any moment m a single one would belong to the actual history of this world if that moment m were actual.

36. As Prior (1967) says, now unpreventable propositions are "those outside our power to make true or false".

37. The notion of analytic implication comes from W. T. Parry (1933) and (1972). See also K. Fine (1986).

38. Suppose the set $\Gamma \subseteq U_a$ contains an atomic proposition u_a predicating an attribute R in a certain order of n objects under concepts. In a propositional logic with attributes, the new set $^*(\Gamma)$ contains three new atomic propositions predicating respectively the historic necessitation $\Box R$, the temporal posteriorization $WillR$ and anteriorization $WasR$ of that attribute of the same objects in the same order. These modal and temporal attributes have the following extensions.

For any $u, \ldots, u_n \in U_c, < u_1, \ldots, u_n > \in \Box R(m/h)$ iff $< u_1, \ldots, u_n > \in R(m'/h')$ for all moments m' coinstantaneous with m and histories h'. Moreover, $< u_1, \ldots, u_n > \in WillR(m/h)$ iff, for at least one $m' > m, <$

$u_1, \ldots, u_n > \in R(m'/h)$. And similarly for $WasR(m/h)$ except that $m' < m$.

39. For the sake of simplicity, I have interpreted the logical constants Tautological and $\geq$ as simple propositional connectives that only rearrange truth conditions rather than secondorder predicates expressing attributes of propositions. According to this interpretation they do not change the structure of constituents. Consequently, all propositional formulas express first order propositions in my formal semantics.

40. Axioms (MT1-2) are A. Zanardo (1985)'s axioms of local correspondence.

41. The present philosophical logic is decidable.

42. See Max J. Cresswell (1975).

43. See his Philosophical Rumifications in Anderson *et al* (1992).

44. I thank Nuel Belnap for having drawn my attention to these laws.

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