

Truth as Modality

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We can analyze the predicate “is true” or the operator “it true that”. The latter functions as a modality. Consequently, we have four principal phrases

- (1) it is true that A;
- (2) it is true that not-A;
- (3) it is not true that A;
- (4) it is not true that not-A.

They form a logical square, analogical to the square for alethic modalities (“it is necessary that A”, etc.). (1) and (2) are contrary, (3) and (4) complementary, (1) implies (3), (2) implies (4), (1) and (4) are contradictory, (2) and (3) are also contradictory.

The square can be extended by adding A, not-A, (1) or (2) (it is true that A or it is true that not-A), and (3) and (4) (it is true that A and it is true that not-A). Clearly, (1) implies A, but (2) implies not-A; the reverse implications do not hold. The formula “(1) or (2)” expresses the principle of bivalence, provided that “is false that” is identified with “it is not true”. The formula “(3) and (4)” is consistent. This means that the principle of bivalence is not a logical law, but a constraint for classical logic.

The consistency of “(3) and (4)” opens the possibility for other logical values than truth or falsehoods, or truth-value gaps. Another interesting fact is that the equivalences of the type “it is true that A if and only if A” (Tarski equivalences) are not logical tautologies. Thus, the expressive power of modal treatment of truth is quite considerable.