Actual infinities in the Tractatus

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Wittgenstein's finitism and Skolem's equational calculus. Largely based on [1], Mathieu Marion famously argued that the Tractatus presents a finitist account of arithmetic. The conception of numbers as "exponents of operations" that we find in aphorisms 6.02-6.031 and 6.22-6.241 of the Tractatus amounts to take arithmetic as a purely syntactic "equational calculus" which could be developed more or less along the lines of Skolem's 1923 paper on the foundations of elementary arithmetic. Given the recursive definitions we find in aphorisms 6.02 (for addition) and 6.241 (for multiplication), we can easily get at definitions which are at the very core of Skolem's system of arithmetic. I claim (and will try to show) that the Tractarian version of Skolem's "equational calculus" has the advantage that the associative law does not have to be "proved" — it is a consequence of the general notion of "operation" on which arithmetical operations are built. The main feature of an "equational calculus" is that it is completely free of quantifiers. General laws (using free variables) are obtained with the help of inductive proofs, and identity is just a sign used to indicate substitutability inside the calculus. It is at least arguable that Wittgenstein will stick to this notion of arithmetic until the end of his life. Finitism in arithmetic could be seen as a constant trait of his philosophy — one that survives the "crisis" undergone by the Tractarian system in the early 30's.

Actual infinities in the Tractatus. Now although I think this is correct of arithmetic, I don't think it is correct in general. I claim it is wrong to say that the Tractatus has no place for the idea of actual infinities. As a matter of fact, it has a doctrine of actual infinities which is completely independent of (and incompatible with) set-theoretic notions. I rest my claim on a careful analysis of aphorism 5.501, where Wittgenstein describes three different methods of "describing" the values of a variable (viz. direct enumeration, the indication of a propositional function, and the indication of a formal law for the construction of propositions). All these methods involve the "description" of propositions, and this notion is problematic from the Tractarian point of view. A proposition is not merely a propositional sign – it is a propositional sign in its projective relation to the world, and this projective relation is not something that could be "described". So our first task is to explain how to interpret the word "description" [Beschreibung] in the aphorism. I will try to show that the word "description" must be taken seriously and that only the "material" or "purely formal" part of a proposition (i.e., the propositional sign itself) must be meant at this point. Then we will have to re-examine the notions of "propositional function" and "formal series" mentioned in the aphorism, and show how these re-examined notions throw a new light on passages like 6.03, where Wittgenstein talks about the "general form of a cardinal number", and uses an "expression in brackets" [Klammerausdruck] to express it. Once we have come to terms with the notion of "description of propositions", we must show that the second and the third methods of selection mentioned in 5.501 necessarily involve an infinite number of propositions. This is not immediately obvious since propositional functions could have a finite number of values, and formal series could just involve the indefinite production of different signs for a finite number of propositions. So we must show that (i) as far as propositional functions are concerned, if they have a finite number of values, the propositional function itself in not necessary for the description of its values; (ii) as regards formal series, if they involve nested quantifiers, then the propositional signs of the series are necessarily associated with an infinite number of different senses. Then we will be prepared to show that, given the logical use of these methods of selection in the Tractatus, the intension (the rule) involved in them is not sufficient. The extension itself must be fully given. So the Tractatus is inevitably committed to the idea of actual (and not merely potential) infinities.

References

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