Connexive logics?

Fabien Schang, National Research University Higher School of Economics
schangfabien@gmail.com

"Connexive" logics? Just as Slater questioned in [3] the alleged invalidity of the Principle of Non-Contradiction (PNC) in paraconsistent logics, the following wants to question the alleged validity of some theses in connexive logics, namely: the so-called Aristotle's Theses (AT) and Boethius' Theses (BT).

Aristotle's First Thesis (AT_1): $\models \neg(\neg\varphi \rightarrow \varphi)$ (AT_1)
Aristotle's Second Thesis (AT_2): $\models \neg(\varphi \rightarrow \neg\varphi)$ (AT_2)

Boethian First Thesis (BT_1): $\models (\varphi \rightarrow \psi) \rightarrow \neg(\varphi \rightarrow \neg\psi)$ (BT_1)
Boethian Second Thesis (BT_2): $\models (\varphi \rightarrow \psi) \rightarrow \neg(\neg\varphi \rightarrow \psi)$ (BT_2)

Classical or material implication is the main target, allowing any false sentence to entail its own negation. Therefore, the task of connexive logic is to validate (AT)-(BT) by revising the properties of two logical constants: negation, and conditional.

Connexive "logics"? Connexive logics are systems in which (AT) and (BT) are tautologies. However, we do not follow this general line. Three different strategies can be followed to make sense of connexivity, as a matter of fact.

(#1) Either (AT) and (BT) are taken to be tautologies.

(#2) Or these formulas are semantic consequences by assuming the Deduction Theorem. Thus:

$$\neg\varphi \models \neg\varphi \quad (AT_{1.1})$$
$$\varphi \models \neg\neg\varphi \quad (AT_{2.1})$$
$$(\varphi \rightarrow \psi) \models \neg(\varphi \rightarrow \neg\psi) \quad (BT_{1.1})$$
$$(\varphi \rightarrow \neg\psi) \models \neg(\varphi \rightarrow \psi) \quad (BT_{2.1})$$

(#3) Or else, (AT) and (BT) are viewed as neither tautologies nor theorems but, rather, as prohibiting inference rules: no negation can be entailed from a given sentence, on the one hand; but, on the other hand, this does not mean that applying sentential negation to these prohibited relation results in a theorem.
The problem is that (#2) and (#3) are supposed to be generally redundant: every "prohibited" formula is a formula whose negation is held to be a theorem. In the following, we endorse only (#3) whilst arguing that the difference between (#3) and (#2) is not trivial.

Connexive logic! First of all, we claim that the main relation of connection takes place, not between sentences, but statements. These are speech-acts of affirmation and denial, depending upon whether the speaker accepts or not the truth (or falsity) of a given sentence.
Following [4], (PNC) can be viewed as a general statement concerning what cannot be affirmed or denied at once. In the same way, (AT) and (BT) claim that one cannot affirm the truth of a given sentence after denying it.
A general semantic framework is used in the following to make sense of the whole debate in connexive logic: Question-Answer Semantics (**QAS**), where meaning results from such a

question-answer game. It relies on a bilateralist theory of judgment: sentential negation is to be expressed in terms of the speech-act of denial (no-answer), rather than the contrary; affirmation and denial are independent sorts of judgment, in the sense that not any denial of a sentence φ entails the affirmation of its sentential negation $\neg\varphi$.

Let us consider a logic of acceptance and rejection, $\mathbf{AR}_4 = \langle L, \mathbf{Q}, \mathbf{A}, f_c, \mathbf{4} \rangle$ [2]. It is a logical system interpreted with a four-valued algebra, such that:

- L is a language including atomic sentences p, q, … and logical connectives between these;
- **Q** is a questioning function associated to a given sentence p: $\mathbf{Q}(p) = \langle \mathbf{q}_1(p), \ldots, \mathbf{q}_2(p) \rangle$;
- **A** is an answering function associated to a given sentence p: $\mathbf{A}(p) = \langle \mathbf{a}_1(p), \ldots, \mathbf{a}_2(p) \rangle$;
- f_c is a set of logical connectives: negation $\neg$, conjunction $\wedge$, disjunction $\vee$, conditional $\rightarrow$;
- **4** is a set of four logical values: 11, 10, 01, 00.

The sentences of $\mathbf{AR}_4$ are to be constructed in accordance to the following Backus-Naur form:

$$\varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi$$

There are n = 2 questions $\mathbf{Q}(p) = \langle \mathbf{q}_1(p), \mathbf{q}_2(p) \rangle$ about a given sentence p, namely: $\mathbf{q}_1(p)$ "Is p told true?", and $\mathbf{q}_2(p)$ "Is p told false?". Each of the four logical values is a single combination of the preceding two questions, namely: $\mathbf{A}(p) = 11$: p is told true, p is told false; $\mathbf{A}(p) = 10$: p is told true, p is not false; 01: p is not told true, p is told false; 00: p is not told true, p is not told false.

Let us note some main properties of this logical system:

(a) Negation as falsity: $\mathbf{a}_1(\neg\varphi) = \mathbf{a}_2(\varphi)$

(b) For every sentential question i,j (about truth or falsity, with i ≠ j):

Inconsistency: not : $\mathbf{a}_i(\varphi) = 1 \Leftrightarrow \mathbf{a}_j(\varphi) = 0$
Coherence: $\mathbf{a}_i(\varphi)\ 1\ x \Leftrightarrow \mathbf{a}_i(\varphi) \neq 0$

(c) For any sentence φ of logical value $\mathbf{A}(\varphi) = \langle \mathbf{a}_1(\varphi), \mathbf{a}_2(\varphi) \rangle$:

Negation: $\mathbf{A}(\neg\varphi) = \langle \mathbf{a}_2(\varphi), \mathbf{a}_1(\varphi) \rangle$
Conditional $\mathbf{A}(\varphi \rightarrow \psi) = \langle \mathbf{a}_1(\varphi) \sqcap \mathbf{a}_1(\psi), \mathbf{a}_1(\varphi) \sqcap \mathbf{a}_2(\psi) \rangle$

The main import of this bilateralist system is the definition of conditional: its truth-conditions are strengthened with respect to the classical version, without collapsing conditional to conjunction.

Application We take the theses of connexive logics to express negative rules, rather than positive ones. In other words, Aristotle and Boethius did not present logical laws in their aforementioned quotations; rather, they talked about what they took to be incompatibilities violating laws of thought.
Thus, (AT) and (BT) are negative rules denying some truth- or falsity-claims, in accordance with the meaning of conditional in $\mathbf{AR}_4$:

$$\mathbf{a}_1(\varphi) = 0 \Rightarrow \mathbf{a}_1(\varphi) \neq 1 \qquad (\mathrm{AT}_{1.2})$$

$$\mathbf{a}_1(\varphi) = 1 \Rightarrow \mathbf{a}_1(\varphi) \neq 0 \qquad (AT_{2.2})$$
$$\mathbf{a}_1(\varphi \rightarrow \psi) = 1 \Rightarrow \mathbf{a}_1(\varphi \rightarrow \neg\psi) = 0 \qquad (BT_{1.2})$$
$$\mathbf{a}_1(\varphi \rightarrow \neg\psi) = 1 \Rightarrow \mathbf{a}_1(\varphi \rightarrow \psi) = 0 \qquad (BT_{2.2})$$

In conclusion, our milder interpretation of (AT) and (BT) fulfills the desiderata of ancient logicians without imposing those of modern logic. This essentially requires a twofold distinction, namely:

- positive (or winning) inference rules (e.g., Modus Ponens), and negative (or non-losing) inference rules (e.g., Modus Tollens) [1]
- locutionary (sentential) negation, and illocutionary negation (denial)

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