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SOME LIAR-LIKE PARADOXES

Resume: This paper formulates some liar-like paradoxes. The basic idea is that self-referentiality, classical logic and T-equivalences are sources of related paradoxes. This background is motivated by Leśniewski's-Tarski's analysis of the classical Liar-antinomy. In particular, paradoxes for pedicates "is meaningless" and "is not rational" are stated.
Keywords: logic, referentiality, T-equivalences, meaningful, rational.

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НЕКОТОРЫЕ ПАРАДОКСЫ, ПОДОБНЫЕ «ЛЖЕЦУ»

Резюме: В этой статье представлены некоторые подобные «Лжецу» парадоксы. Основная идея состоит в том, что их источником являются самореферентность, классический характер логики и эквивалентности, связанные с предикатом истинности Т. В основе такого понимания лежит анализ классической антиномии «Лжец» у Лесневского и Тарского. В частности, представлены парадоксы, возникающие в связи с предикатами «быть лишённым значения» и «не быть рациональным».
Ключевые слова: логика, референциальность, Т-эквивалентности, значимость, рациональность.

The classical Liar Paradox (LP, for brevity) runs as follows (it is a slightly modified version proposed in Poland by Jan Łukasiewicz and employed by Alfred Tarski):

(1) the sentence denoted by (1) is false;
(2) (1) $\Leftrightarrow$ (1) is true;
(3) The sentence (1) is false $\Leftrightarrow$ The sentence (1) is true;
Contradiction!

Remark: How to understand the premise (2)? Some commentators say that it is a nonsense, because it assume the equality (1) = (1) is true, but this equation is plainly false, if identity is understand in its official logical meaning. Imagine, however, that we correlate some objects with numbers (numerals), for example, say that Lionel Messi has the number 10 as a soccer player in FC Barcelona or the Argentinian national team. Conse-

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quently, we are entitled to say 10 = Lionel Messi in such and such context. The same can be said in the case of LP. What is important, is that the equivalence (b) is justifies by this convention.

Stanisław Leśniewski, followed by Tarski, offered a diagnosis of Leśniewski–Tarski. According to them, we can identify three sources of LP

(i) Self-referentiality of the predicate "is false";
(ii) T-scheme, that is, the formula *A* is true $\Leftrightarrow$ *A*;
(iii) Classical logic.

This diagnosis opens related possibilities concerning of how avoid LP. The first consists in rejecting self-reference of semantic predicates, the second — banishes T-equivalences, and the third goes by changing logic. According to Tarski, T-scheme is too intuitive to be rejected and classical logic should be protected as the best. Consequently, rejecting self-reference of semantic predicates leads to the least theoretical costs. A natural outcome of the first strategy consists in the language (L) / metalanguage (ML) distinction. An important lesson stems from the above analysis: nothing is free of charge, also in logic.

Remark. One can ask whether elimination of self-reference concerns only the predicate "is false". What about the predicate "is true"? Call both as aletheiological words. It was proved that the formula

(*) (1) $\Leftrightarrow$ (1)

is true, does not lead to any paradox. However, if we change logic into so-called dual logic (or other in which the logical value of falsehood is distinguished; *A* is a dual tautology if and only if it is false for any valuation) (*) leads to the dual LP or the Truth-teller paradox (see [Woleński 1995] for details). Thus, both aleteiological predicates produce semantic troubles.

Remark. Changing logic does not block the strengthened LP. It is captured by

(a) (1) the sentences (1) is not true (false or other);
(b) (1) $\Leftrightarrow$ (1) is true;
(c) (1) is true $\Leftrightarrow$ (1) is not true;
Contradiction!

If (1) is *n* (other than false or true), we get (1) is *n* $\Leftrightarrow$ *n* is true, which also cannot be accepted.

A very important observation is this. T-scheme plays the fundamental role in deriving LP (if all of its forms). On on the other hand there not paradoxical cases, Consider an anthropomorphic sentences that say "I am provable" or "I consists of 3 words". Clearly, we cannot formulate relevant T-schemes. The relation of truth and provability is captured by Löb's theorem, that is, the formula "if *A* is provable, *A* is true". The reverse implication does not hold, due to the Tarski undefinability theorem. Turner [Turner 1990; Turner 1990a] observed that the only-part of T-equivalence, that is, formula "$A \Rightarrow A$ is true", is responsible for LP. Contrary to important uses of the relation between truth and provability in metamathematics, any attempt to form a T-sentence for "I consists of 3 words" (on the than falling under the scheme "*A* is true if and only if *A*") leads to an obvious

absurdity, because omitting "consists of 3 words" converts the entire sentence into a nominal expression if we decide to treat *A* as an abbreviation for "*A*" or reduces this sentences to itself.

Are "is true" and "is false" the only predicates, which produce paradoxes similar to LP? The answer is negative and I will show that some other predicates share the fate of Historically speaking, self-referential use of the "is meaningful" was suspected to be troublesome. This problem arose as connected with the question "Is verifiability principles as proposed by logical empiricism, meaningful or nor?" (see [Ingarden 1936; Ewing 1937; Woleński 1990; Woleński 2010]). Logical empiricists argued that we should accept

(PV) A sentence *A* is meaningful if and only if *A* verifiable or tautological.

Ingarden and Ewing asked whether (PV) itself is verifiable or tautological; I neglect difference between both philosophers in their criticism in question. According to Ingarden and Ewing, both alternatives are untenable, because if it is difficult to imagine how (PV) could be verified by empirical data, but, on the other hand, if this principle is a tautology it is devoid of meaning. Consequently, (PV) should be considered as devoid of meaning by own criteria or, as Ingarden pointed out, the principle in question smuggles a hidden meaning to metalogic and thereby remains unclear.

Carnap briefly commented Ingarden's criticism at the Prague Congress in 1934 and, coherently with his treatment of logic as syntax, argued that metalogical sentences belong to pure syntax and, thereby, they do not require empirical verification [Carnap 1936]. Consequently, metalogical sentences are tautological just by definition and it means that their validity can be established by formal combinatorial procedures. Moreover, Carnap stressed that one of the most crucial difference between logical empiricism and phenomenology consists in admitting *Wesenschau* by the latter school and rejecting this kind of cognition by the former philosophical group. Although Ingarden could explain the hidden meaning of metalogical sentences by recurring to phenomenological intuition, but nothing essentially depends in Ingarden criticism on a specific basis of the hidden meaning. It is sufficient to observe that metalogical sentences are meaningful by virtue of other criteria than stated by (PV). Both parties can also agree that empirical verification has nothing special to do in metalogic. In fact, Carnap and other logical empiricists accepted this position and looked for a solution via the concept of analyticity. Pap in his comments on Ewing 1937 [Pap 1957] proposed a way out consisting in considering (PV) as a result of an analysis of the concept of meaning. Pap remarked that this view overcomes the objection that (PV) says nothing at all as being a logical tautology. Actually, this proposal weakens Carnap's original idea that metalogical assertions

In order to formulate, the Liar-like paradox for "is meaningful", we need two following additional principle:

(4) *A* is meaningful $\Leftrightarrow \neg A$ is meaningful;
(5) *A* is meaningful if and only if *A* is true or false.

The principle (4) says that meaningfulness is invariant under negation. Its justification can appeal to the idea that purely logical operations do not influence the logical status of their arguments. Consequently, if *A* is logical sentence (a tautology or contradiction), its denial is tautology or contradiction s well, and, if *A* is factual (neither a tautology

or contradiction), its negation is factual as well. Yet negation changes logical value, (5) exhibits the form of meaningfulness related to classical logic.

Having the statements (4) and (5), we proceed in the following way. Firstly, we for a sentence with the self-referential employment of the predicate "is not meaningful". This step is given by

(6) (6) is not meaningful (assumption).
(7) ¬(6) is not meaningful (by (4)).

On the other hand, (7) says that (6) is meaningful. Consequently, (6) is either true or false (by (5). Assume that (6) is true. It gives

(8) (6) is true ⇔ (6) (an instance of T-scheme).

By the detachment, we obtain (6), but it produces a contradiction, because (6) appears as true and not meaningful.

Assume that (6) is false. Hence, (6) is meaningful, but ¬(6) jest meaningful and true. If so, we have

(9) ¬(6) ⇔ (6) is meaningful.

As a consequence we obtain

(10) (6) ⇔ (6) is not meaningful.

However, this step convert us to the former case, because it implies that (6) is meaningful if and only if it is true.

Remarks. If we say that being not meaningful covers a third value, we obtain the strengthened meaningfulness paradox. Ewing [Ewing 1937] observed another difficulty. Consider (PV) and ask what happens if we check its truth or falsity without taking into account sentences like (6) that is asserting own meaninglessness. Clearly, (PV) should be meaningful under its truth as well as under its falsity. However, the latter assumption leads to a strange consequence that (PV) is false and meaningful, but in such a case it defeat itself. It is not a logical paradox because not all eventualities lead to formal inconsistency (more specifically, it is the case of working under the assumption that (PV) is true), but a plain pragmatic or epistemological oddity. Another strange consequence follows from inspecting (7). The negation of (6) means that (6) is meaningful. If so, asserting (7) commits us to the view that (6) is meaningful, provided that it is not.

Another example of a Liar-like paradox concerns the concept rationality. As an example I consider what can called Cartesian rationality defined by

(11) (a) *A* is rational iff its truth is dictated the infallible method;
(b) *A* cannot be rational and false.

We can form the self-referential sentence

(12) (12) is not rational (its truth is not dictated by the infallible method),

which leads to

(13) (12) is true $\Leftrightarrow$ (12) is not rational.

By propositional calculus, (12) either true by the infallible method and not rational or false and rational. However, both possibilities are contradictory. Presumably, one could thing that other definitions of rationality suffer from the paradox. Unfortunately, it is not so. Let us adopt

(14) *A* is rational $\Leftrightarrow$ *C*,

where the letter *C* refers to a condition of rationality. Thus, we have

(15) *C* is rational $\Leftrightarrow$ *C*;
(16) C is rational $\Leftrightarrow$ *C* is true.

Now, if we consider

(17) *C* does not satisfy the condition *C*,

we obtain a version of a Liar-like paradox. The situation can be easily generalized for such predicated as "it is analytic", "it is confirmed", "it is scientific", etc. A hypothesis can be stated that almost every interesting epistemological predicate falling under the scheme

(18) *A* is *X* $\Leftrightarrow$ *C* ($\Leftrightarrow$ *C* is true),

leads to a Liar-like paradox. (18) confirms that the role of T-scheme in Liar-like paradoxes is crucial. Perhaps the predicate "is dubious" appear as particularly interesting, because it is related to the issue of skepticism.

A general situation looks as follows. If a given principle P establishes the meaning of a predicate *P* referring to properties of sentences such that T-scheme is applicable, we can expect that the predicate in question can generate a Liar-like paradox. In order to prove that *P* does not produce paradox, one must prove that "is *P*" and "is true" do not coincide. It is the case of "is provable". Taking a lesson from metamathematics, *P* is not paradoxical, if there is a procedure, which allows to embed this predicate into the object-language via a procedure analogical to arithmetization. Unfortunately, it is unclear how typical epistemological predicates could converted into the object language.

However, it does not mean that philosophers must resign from *P.* Generalizing the case of using "is true" in order to avoid the Liar-paradox, we can decide that *P* is formulated in ML and apply to items formulated in L. Yet this simple prescription leads to far-reaching metaphilosophical consequences. Take, for example, the problem of how science and philosophy are mutually related. The claim that philosophy (in our case, epistemology) is one of sciences, requires an assumption that (PV) and similar principles are embeddable into L as it is used in science. As we know this assumption is not obvious. I will this fundamental problem without further considerations. Still one remark is in order. Pap's view that (PV) offers an analysis of the concept of meaning leads, as I already remarked, to the question "What about the status of this principle and similar ones?"

A preliminary and very tentative answer is that such analytical products are analytical statements, not reducible to tautologies of LM. Otherwise speaking, if *C* is a criterion of analyticity in L, other (in fact, extended) understanding of "is analytic" functions in ML (see [Woleński 2004] for some proposals in this respect).

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