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NON-DISTRIBUTIVE IMPLICATIONS AND THEIR APPLICATION IN QUANTUM THEORY

In classical situations, which deal with the macro events, the uncertainty is described by the means of the Kolmogorov probability theory that coincides with classical logic and allows specification of conditional probabilities of the poster events with respect to the probabilities of the priory events and entailment errors [7]. In contrast, in micro world, which follows the Heisenberg uncertainty principle, the laws that govern the uncertain decisions in the framework of classical logic differ from the Kolmogorov probabilistic laws [4].

The attempts to agree the Kolmogorov probability with classical logic applied to the quantum mechanical systems resulted in several interpretations of quantum mechanics, the most of which either failed or cannot be considered as meaningful physical rather than pure mathematical interpretations. To resolve this inconsistency, in the studies originated by philosophical conjectures of Popper and von Mises it was suggested revising the classical concepts of probability in their application to quantum world, and in the studies that follows the Heidegger philosophy the revision was focused on the logic that is used in quantum mechanical implications [2]. The last approach produced a wide range of studies that resulted in different variants of the logic of quantum mechanics and related logical calculi [4]. Finally, in their recent work [3] De Raedt, Katsnelson and Michielsen suggested a combined approach, following which quantum mechanical experiments should be considered as "generators" of certain probabilities, while the logic used in quantum mechanical inferences should follow the steady case decision-making led by these probabilities.

However, the mechanism that governs the definition of the probabilities with respect to logical operations and, backward, the probability laws underlying the logic of quantum mechanics are still unclear. The proposed work is aimed to bridge this gap using recently discovered probability-based logical aggregators [8-10]; preliminary results obtained in this direction support a usefulness of such techniques for achieving the indicated aim.

Let a, b and c be observations of the events A, B and C, respectively, and denote by $\wedge$ and $\vee$ general conjunction and disjunction operators. The formal difference between classical and quantum logics is that if the values a, b and c are associated with the truth values from the interval $[0,1]$, then in the quantum logic the distributive law is broken down and is substituted by the inequality $a \wedge (b \vee c) > (a \wedge b) \vee (a \wedge c)$ [2]. In order to satisfy this inequality and to preserve the unified consideration of the classical and quantum events, in the proposed approach it is assumed that both in macro and micro worlds the truth values a, b and c have the same meaning and their underlying probabilities are still Kolmogorov, while the indicated inequality is satisfied by the appropriate definition of the aggregating operators that extend the operators $\wedge$ and $\vee$.

The extension of the Boolean logical operators is obtained using the uninorm aggregator $\oplus_\theta : (0,1) \times (0,1) \to (0,1)$ with neutral element $\theta \in (0,1)$ [10] and the absorbing norm aggregator $\otimes_\vartheta: (0,1) \times (0,1) \to (0,1)$ with absorbing element $\vartheta \in (0,1)$ [1] both defined using the probability distributions as the inverses of their generating functions $u: (0,1) \to (-\infty, \infty)$ and $v: (0,1) \to (-\infty, \infty)$ [6]. Then, the system $\mathcal{A} = \langle (0,1), \oplus_\theta, \otimes_\vartheta \rangle$ forms a non-distributive algebra, in which the distributivity depends on the distributions u^{-1} and v^{-1} and their parameters θ and ϑ and for either for certain classes of distributions it holds true that $a \otimes_\vartheta (b \oplus_\theta c) > (a \otimes_\vartheta b) \oplus_\theta (a \otimes_\vartheta c)$. Finally, since the aggregators $\oplus_\theta$ and $\otimes_\vartheta$ are unambiguously correspond to the general conjunction $\wedge$ and disjunction $\vee$ operators, the required inequality is obtained.

The interpretation of the algebra $\mathcal{A}$ follows the Birkhoff and von Neumann approach [2]. As physical quantities, the values $a, b, c, \ldots \in (0,1)$, on which act the algebraic operators $\oplus_\theta$ and $\otimes_\vartheta$, are considered as observation results, while these values as logical truth values with the operators $\oplus_\theta$ and $\otimes_\vartheta$ considered as logical aggregators define the implication rules that govern the observer's conjectures. In addition, it is expected that the probability distributions that generate the indicated aggregators will be equivalent to the probability distributions, which define the inference probabilities introduced by De Raedt, Katsnelson and Michielsen [3]

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