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FICTIONALISM, CONSTRUCTIVE EMPIRICISM, AND THE SEMANTICS OF MATHEMATICAL LANGUAGE

The biggest challenge to fictionalism lies in its claim that mathematical statements are false by nature. By the indispensability argument, it is highly implausible to argue that the mathematics we use in so many different fields that are physically observable is false. Fictionalists argue that abstract mathematical objects only exist in the context of the "story of mathematics," and so strictly speaking, they do not exist in the real world. Therefore, when a fictionalist asserts a mathematical statement, he only believes the statement is true relative to the specified contextual setting. By contrast, the indispensability argument suggests that, since these seemingly abstract objects are irreplaceable to our best, testable scientific theories, it must follow that these objects actually exist in the real world. Field believes that the indispensability argument is the only good argument for platonism against fictionalism.

In trying to weaken the indispensability argument, and hence one of the principal justifications for mathematical realism, Field attempts to show the dispensability of mathematics in scientific theories. He provides the example of Newtonian gravitational theory, where, without using the standard mathematics, he is able to derive comparable results nominalistically. While Field does not advocate stripping scientific theories of their mathematical content, the example demonstrates the potential dispensability of mathematical objects to scientific theories, and this alone is enough to weaken the indispensability argument, at least supposing the (obviously highly disputed) validity of Field's results in *Science Without Numbers*. To explain his "dispensability" argument, Field claims that mathematics is conservative. This means that combining a mathematical theory and a nominalistic scientific theory does not yield nominalist consequences that could not be derived within the nominalistic scientific theory alone. If this conservativeness result is sound, it follows that the fictionalist view of mathematics does not affect scientific practice. Therefore, adequately justifying the conservative quality of mathematics is crucial for the fictionalist refutation of the indispensability argument.

Field also offers an intuitive argument for conservativeness, suggesting that a "bad" mathematical theory would be subject to revision depending on its conservativeness, not on its truth-value. For example, if a mathematical theory implied statements about history or biology, people would be very skeptical of that mathematical theory. Even though the statements about history and biology may be true, people still remain suspicious of the mathematical theory because the theory is not conservative. Hence, Field arrives at the conclusion that good mathematics need not be true, but it must be conservative.

This paper proposes treating Field's fictionalism in the philosophy of mathematics within the trajectory of van Fraassen's constructive empiricism in the philosophy of science. In his constructive empiricism, van Fraassen places great emphasis on the possibility of varying degrees of theory acceptance. He claims that, when a scientist accepts a theory, the scientist does not need to believe in its contents in a metaphysical sense, nor assert it outright. Van Fraassen's argument is partly motivated by the fact that he does not believe, in a metaphysical sense, in the unobservable objects in scientific theories. Van Fraassen's main reasoning, however, is that the goal of science is not to reach the absolute truth, but to arrive at the truth about the observable aspects of the world. Indeed, if van Fraassen's understanding of the goal of science were true, then good scientific theories would only have to be empirically adequate, meaning it would not necessary for the theory to be true in a metaphysical sense.

In Field's fictionalism, good mathematical theories do not need to be true, but rather must be consistent and conservative. Likewise, Van Fraassen views science to be nothing more than a

study of obtaining truths about the observable phenomena of the world, so good scientific theories need not be true, merely empirically adequate. The takeaway from this comparison is that the concepts of acceptance and empirical adequacy of constructive empiricism can be used to better understand fictionalism. At first glance, constructive empiricism does not seem to help the case of fictionalism because the constructive empiricism is based on the possibility of empirical verification, which is also the basis of indispensability argument. This paper argues, however, that the empirical adequacy of scientific theories is comparable to the conservative nature of mathematical theories. By understanding that a scientific theory need not be entirely true, the falsity of mathematical statements in the fictionalist view becomes more graspable.