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NATURAL DEDUCTION FOR KLEENE'S THREE- AND FOUR-VALUED LOGICS

Regular three-valued logics were presented by S.C. Kleene [3] in to order to develop notation for ordinal numbers as well as to study recursion theory. Connectives of these logics produce partial recursive predicates. The third truth value is understood as "undefined". E.Y. Komendantskaya showed that there are four pairs of regular three-valued conjunctions and disjunctions and one regular three-valued negation. Changing the set of designated values, we obtain 8 logics. One of them is the well-known Kleene's strong logic $\mathbf{K_3}$. Another one is $\mathbf{K_3}$'s counterpart with two designated values that is Asenjo-Priest's [1, 8] logic of paradox $\mathbf{LP}$ ($\mathbf{K_3^2}$). Natural deduction systems for $\mathbf{K_3}$ and $\mathbf{LP}$ were introduced by Priest [8]. One of the purposes of this report is to present, following [5], natural deduction systems for all the other regular three-valued logics.

A process of generalization of Kleene's three-valued logics to the four-valued case was started by M. Fitting [2]. Taking his inspiration from the theory of bilattices, he presented several four-valued analogues of Kleene's three-valued logics. The secondary aim of this report is to present, following [6], natural deduction systems for them.

N.E. Tomova [9] calculated that there are 6400 pairs of regular four-valued conjunctions and disjunctions as well as one regular four-valued negation. Moreover, as follows from [9], only six of these pairs are also monotonic. Note that in the four-valued case, in contrast to the three-valued one, the classes of monotonic and regular logics do not coincide. Note also that Tomova's four-valued generalizations of Kleene's logics differ from Fitting's one. One more purpose of this report is to present, following [7], natural deduction systems for these both monotonic and regular logics.

Thus, in this report, we will discuss axiomatization of all Kleene's three-valued logics as well as all their four-valued generalizations which are known from the literature for the current moment.

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