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ЛОГИКА СЕГОДНЯ

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WHAT OUGHT TO BE DONE FOR IMPERATIVE LOGIC

Abstract. This paper presents the logical treatment of imperatives (commands and promises) regarded as actions one rational agent performs in order to compel the other rational agent, by imposing certain obligations on her, to act in a certain way. To talk about the embedded *stit*-formulas that represent such imperative actions the framework of *stit*-logic supplied with the $\bigcirc$-operator, as it is introduced by J. Horty, is used. We investigate some curious technical details of using the $\bigcirc$-operator ranging over the set of agents along with different *stit*-operators and suggest formulas picturing some properties typical for the usage of commands and promises. We establish several theorems and propositions expressing essential principles of imperative agency and show that these principles differ substantially from their analogs in propositional logic and standard deontic logic (SDL). Finally, we briefly discuss the possibilities of further investigation that embodies such issues as expressing the imperative permission and prevalent embedded imperatives in the given framework.
Keywords: STIT-logic, logic of action, agency, imperative, obligation.

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ЧТО НУЖНО СДЕЛАТЬ ДЛЯ ЛОГИКИ ИМПЕРАТИВОВ[2]

Аннотация. В статье представлен логический подход к императивам (командам и обещаниям), которые трактуются как действия одного рационального агента, направленные на то, чтобы побудить другого рационального агента, посредством связывания его обязательствами определенного рода, к совершению некоторого поступка. Для того, чтобы иметь возможность интерпретировать формулы с вложенными агентными операторами, которые отражают таким образом понимаемые императивы, используется STIT-логика, дополненная деонтическим оператором «O» хортиевского типа. В статье исследуется ряд любопытных технических деталей, связанных с использованием деонтического оператора в рамках логики действия совместно с различными агентными операторами, и предлагаются формулы, отражающие характеристические свойства использования команд и обещаний. Мы формулируем несколько теорем и наблюдений, отражающих существенные свойства употребления императивов, и показываем, что между ними и сходными с ними утверждениями из пропозициональной и стандартной

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деонтической логики отсутствует параллелизм. В заключении мы кратко обсуждаем перспективы будущих исследований в этой области, касающихся представления императивного разрешения и широко рапространенных вложенных императивных действий.
Ключевые слова: STIT-логика, логика действий, агентность, императив, обязательство.

Motivation

Only few philosophers and an even fewer number of logicians focus their attention on imperative sentences. Probably the most famous contribution was made by J. L. Austin in his studies on commands, promises, requests and other types of speech acts (Austin 1975). Foundations of illocutionary logic that were presented in the treatise of the same title by J. R. Searle and D. Vanderveken (Searle, Vanderveken 1985), almost 25 years after Austin, deserve less recognition. Several attempts to revive interest to imperatives have been made recently (see papers (Vranas 2008), (Kearns 2006) and (Parsons 2013)).

However, imperatives play a great role in our communication activity, and logic should say something about this fact. Just in the same way it says a lot of essential things about declaratives. Being more specific, we believe that logic should establish some rules that govern the usage of imperatives, above all—the usage of commands and promises, since these types of imperatives are the prevaling subjects of controversy in a great many of situations, starting from some ethical questions and ending with argumentation that each of us uses permanently.

Since it is quite natural to consider commands and promises as actions that agents perform in order to impel other agents to act in a certain way, we can state that in order to contribute to the study of imperatives logic needs to learn how to speak about these kinds of actions. Fortunately, logic has already known it, and we have a lot of logic of actions at our disposal. What we need now is to choose one of them and adopt its language for expressing actions of commands and promises.

Stit-logic or the logic of *seeing-to-it-that* is mostly known from the series of works published in quantity by N. Belnap, M. Perloff and M. Xu in the period from 1991 to 1995, looks quite attractive as a basis for the formal study of imperative actions. And it is definitely so, since the action of a command or a promise can be grasped rather naturally at first glance by means of its main formula

$$[\alpha \textit{ sees to it that}\colon \phi], \tag{1}$$

where α stands for an agent, and ϕ stands for a particular situation that α guarantees to be the case, as it was first mentioned by Perloff in (Perloff 1995). A more precise way of picturing an imperative, which generally includes a pair of agents, might be this:

$$[\alpha \textit{ sees to it that}\colon [\beta \textit{ sees to it that}\colon \phi]]. \tag{2}$$

What follows investigates this proposal in a formal way using the language and the branching time semantics (BTS) of *stit*-logic.

The article consists of six sections. Section 1 provides a brief explanation of the basic formal tools that are used in the analysis. Those who are familiar with articles on *stit*-logic can skip it. Section 2 introduces and explains the main difficulty of applying the machinery of *stit*-logic to imperatives and overviews some possible solutions. Here the main formula for picturing imperatives is introduced and some preparatory semantic considerations are stated. The next three sections contain proofs of some theses which express the essential logical features of commands and promises and give answers to some old puzzles of imperative logic: Section 3 speaks mainly on commands—here the refraining principle with respect to command is established, and some divergence between imperative logic with $\bigcirc$-operator and Standard Deontic Logic is elicited; in Section the 4 indexed $\bigcirc$-operator is introduced and its expressive power in the case of promises is demonstrated; in Section 5 the question of complex imperatives is raised over again and is answered from the perspective elaborated in the article. The last Section 6 shows some perspectives for the future investigations of imperative actions.

1 The basic language $\mathcal{L}$ of *stit*-logic and the BTS

Definition 1.1 (language $\mathcal{L}$ of *stit*-logic)**.** *Let* Φ *be the set of proposition letters* p, q, r, $\ldots$, *and* Γ *be the set of agent terms* α, β, γ, *...Well-formed formulas* ϕ *are formed in accordance with the following rule:*

$$\phi := p \mid \neg\phi \mid \phi \wedge \phi \mid \Diamond\phi \mid [\alpha\ astit\colon \phi] \mid [\alpha\ dstit\colon \phi],$$

where p *ranges over elements of* Φ, *and* α *ranges over elements of* Γ.

Here $\Diamond$ serves as an analog for *Poss*-operator, which designates historical possibility. The historical necessity operator $\Box$ (in *stit*-theory "*Sett*" is sometimes used) is dual for $\Diamond$ as usual.

Definition 1.2 (frame $\mathcal{S}$)**.** *A frame* $\mathcal{S}$ *for the language* $\mathcal{L}$ *is a tuple* $\langle \boldsymbol{T}, \leq, \Gamma, \boldsymbol{Choice}\rangle$, *where* $\boldsymbol{T}$ *is a nonempty set of moments* $m_1, m_2, \ldots, m_n$; $\leq$ *is a partial order on* $\boldsymbol{T}$; Γ *is a set of agents* α, β, γ, *...;* ***Choice*** *is a function such that for each agent* $\alpha \in \Gamma$ *and for each moment* $m \in \boldsymbol{T}$, $Choice^{\alpha}_{m}$ *is the set of possible choices available to* α *at* m.

A history $h \in \mathbf{T}$ is a maximal set of moments ordered linearly. $H_{(m)}$ is a set of histories passing through m, and thus $Choice^{\alpha}_{m}$ is a partition of $H_{(m)}$. $Choice^{\alpha}_{m}(h) \in Choice^{\alpha}_{m}$ is a particular choice that α does in m.[3] All histories that are in $Choice^{\alpha}_{m}(h)$ are called choice-equivalent histories for α at m. If h_1 and h_2 are in $Choice^{\alpha}_{m}(h)$ we write that $h_1 \equiv^{\alpha}_{m} h_2$. $\mathbf{T}$ can be regarded as the set of sets $i_1, i_2, \ldots, i_n$ of moments called *instants*, where for every moments $m \cup m' \in i$, $m \not\leq m'$ and $m' \not\leq m$. If $m_1 \cup m_2 \in i$, $m_1 \in h_1$ and $m_2 \in h_2$, and $h_1 \equiv^{\alpha}_{m} h_2$, then m_1 and m_2 are choice-equivalent moments for α at m. $i_{(m)}$ designates instant that contains moment m. If

[3]Sometimes we designate such particular choice-cell as $\mathbf{K}_1, \mathbf{K}_2, \ldots, \mathbf{K}_n$ following Horty's notation.

some histories h_1 and h_2 are *undivided* at some moment m (which means that there is a moment $m' \in h_1 \cup h_2$ such that $m < m'$) then there is no choice between h_1 and h_2 for any agent at m.

Definition 1.3 (model $\mathcal{M}$). *A model $\mathcal{M}$ for the language $\mathcal{L}$ is a tuple $\langle \mathcal{S}, v \rangle$, where $\mathcal{S}$ is a frame for the language $\mathcal{L}$, and v is a function that assigns values to atomic formulas in each index m/h.*

$\mathcal{M}, m/h \models p$ iff $m/h \in v(p)$.

$\mathcal{M}, m/h \models \neg\phi$ iff $\mathcal{M}, m/h \not\models \phi$.

$\mathcal{M}, m/h \models \phi \wedge \phi$ iff $\mathcal{M}, m/h \models \phi$ and $\mathcal{M}, m/h \models \phi$.

$\mathcal{M}, m/h \models \Diamond\phi$ iff $\mathcal{M}, m/h_1 \models \phi$ for some $h_1 \in H_{(m)}$.

$\mathcal{M}, m/h \models [\alpha\ astit\colon \phi]$ iff there is a moment $w < m$ such that $\mathcal{M}, m_1/h_1 \models \phi$ for every $m_1 \equiv^{\alpha}_{w} m$ and every $h_1 \in H_{(m_1)}$ (the positive condition), and $\mathcal{M}, m_2/h_2 \not\models \phi$ for some $m_2 \in i_{(m)}$ and for some $h_2 \in H_{(m_2)}$ (the negative condition).

$\mathcal{M}, m/h \models [\alpha\ dstit\colon \phi]$ iff $\mathcal{M}, m/h' \models \phi$ for every $h' \in Choice^{\alpha}_{m}(h)$ (the positive condition); and $\mathcal{M}, m/h'' \not\models \phi$ for some $h'' \in H_{(m)}$ (the negative condition).

We say that a formula $[\alpha\ astit\colon \phi]$ is *settled true* at m ($\mathcal{M}, m \models [\alpha\ astit\colon \phi]$) iff $\mathcal{M}, m/h \models [\alpha\ astit\colon \phi]$ for every $h \in H_{(m)}$. A formula $[\alpha\ astit\colon \phi]$ is valid in $\mathcal{M}$ ($\mathcal{M} \models [\alpha\ astit\colon \phi]$) iff $\mathcal{M}, m \models [\alpha\ astit\colon \phi]$ for every m from $\mathcal{M}$. A formula $[\alpha\ astit\colon \phi]$ is valid in $\mathcal{S}$ ($\mathcal{S} \models [\alpha\ astit\colon \phi]$) iff $\mathcal{M} \models [\alpha\ astit\colon \phi]$ for every $\mathcal{M}$. A formula $[\alpha\ astit\colon \phi]$ is valid ($\models [\alpha\ astit\colon \phi]$) iff $\mathcal{S} \models [\alpha\ astit\colon \phi]$.

The same holds with respect to formulas with *dstit*-operator, except the fact that *dstit*-formulas cannot be settled true: $\mathcal{M}, m \not\models [\alpha\ dstit\colon \phi]$, since the opposite contradicts the negative condition for *dstit*-formula from the definition 1.3.

As usual an arbitrary formula ϕ is satisfiable if there is a model $\mathcal{M}$ and an index m/h such that $\mathcal{M}, m/h \models \phi$.

2 *Stit*-theory offers the problem

Since the language $\mathcal{L}$ allows nesting we could try to capture formally the proposal contained in expression 2. However the nested *stit*-formula that engages *astit*- and *dstit*-operators turns out to be unsatisfiable.

Proposition 2.1. *$\mathcal{M}, m/h \not\models [\alpha\ astit\colon [\beta\ dstit\colon \phi]]$ for every model $\mathcal{M}$ and every index m/h.*

Proof. Suppose there is an arbitrary model $\mathcal{M}$ and an arbitrary index m/h such that $\mathcal{M}, m/h \models [\alpha\ astit\colon [\beta\ dstit\colon \phi]]$. Then there is a moment $w < m$ such that both positive and negative requirements for $[\alpha\ astit\colon [\beta\ dstit\colon \phi]]$ are fulfilled. Then, in particular, $\mathcal{M}, m_1/h_1 \models [\beta\ dstit\colon \phi]$ for every $m_1 \equiv^{\alpha}_{w} m$ and every $h_1 \in H_{(m_1)}$. This means that the negative condition for $[\beta\ dstit\colon \phi]$ cannot be fulfilled, since if it is the case, then there would be an index $m_1/h'_1 \not\models \phi$, and $\mathcal{M}, m_1/h_1 \models [\beta\ dstit\colon \phi]$ can no longer hold. □

The semantics of *stit*-theory puts constraints on nested *stit*-formulas, and to express actions of commands and promises in $\mathcal{L}$ we should somehow evade this constraints. This difficulty was familiar to Belnap's team: in their joint proceeding (Belnap, Perloff, Xu, 2001) (especially Ch. 4 and Ch. 12) we can find a number of its possible solutions, though presented mostly in an informal manner and in a general way. On the whole these proposals come to an idea of some complement that should be introduced to the nested *stit*-formula and thus prevent its self-contradictoriness. The most popular types of complements were these:

- deontic ($[\alpha\ stit\colon Obligatory\ [\beta\ stit\colon \phi]]$, $[\alpha\ stit\colon Permissible\ [\beta\ stit\colon \phi]]$);[4]
- modal auxiliary verbs ($[\alpha\ stit\colon Should\ [\beta\ stit\colon \phi]]$, etc.);
- illocutionary complement ($[\alpha\ stit\colon Advises\ [\beta\ stit\colon \phi]]$, $[\alpha\ stit\colon Orders\ [\beta\ stit\colon \phi]]$).[5]

We find the idea of using deontic operators wedging into nested *stit*-formulas quite attractive. Not only may it preserve the acceptability of the view on imperatives as actions that produce other actions, but also because it explicates the fact that imperatives issued deontically influence the engaged agents. The last thing agrees with CUGO- and PUGO-principles introduced in (Yamada 2008), which say respectively that **C**ommands **U**sually **G**enerate **O**bligations and **P**romises **U**sually **G**enerate **O**bligations. In accordance with this we state that

$$[\alpha\ stit\colon Obligatory\ [\beta\ stit\colon \phi]] \tag{3}$$

pictures the situation that when agent α performs an action of a command and addresses it to agent β, α thereby creates an obligation, which is either fulfilled or not fulfilled by β's future course of action. The following study of logical properties of formula 3 and formulas that can be obtained from it through common formation rules is of course on no account the direct study of imperatives, but rather it is the study of deontic effects of imperative sentences regarded as actions, maybe the most important type of effect the actions of commands and promises have.

2.1 Nested *Stit*-formulas with ought-operator

Though we do not find the formal treatment of formulas like

$$[\alpha\ stit\colon Obligatory\ [\beta\ stit\colon \phi]]$$

in (Belnap, Perloff, Xu, 2001), we can still find the semantic rules for its complement in the earlier proceeding (Horty, Belnap 1995). There the impersonal *ought to be*-operator "$\bigcirc$" and *dstit*-operator are regarded as a unit so that it becomes possible

[4]From this point onwards we use "*stit*" as a generic term for operators *astit* and *dstit*.

[5]It is worth mentioning that even the introduction of the historical possibility operator $\Diamond$ to the nested *stit*-formula preserves it from being self-contradictory, although it does not make $[\alpha\ stit\colon \Diamond[\beta\ stit\colon \phi]]$ a relevant picture of a command or request.

to express in terms of *stit*-theory such deontic notion as personal *ought to do*. Then the formula

$$\bigcirc\,[\alpha\ dstit\colon \phi] \tag{4}$$

pictures the fact that agent α ought to see to it that ϕ, or that agent α is obligated to see to it that ϕ.

In order to provide the semantics for formula 4 the frame $\mathcal{S}$ is extended in the following way:

Definition 2.2 (extended frame $\mathcal{S}^+$). *An extended frame $\mathcal{S}^+$ for the language $\mathcal{L}^+$ is a tuple $\mathcal{S}^+ = \langle \boldsymbol{T}, \leq, \Gamma, Choice, Ought\rangle$, where $\boldsymbol{T}$, $\leq$, Γ and $Choice$ are defined as in the definition 1.2, and where $Ought$ stands for the function mapping each moment m into the set $Ought_{(m)} \in H_m$ of ideal histories passing through moment m.*

The language $\mathcal{L}^+$ is obtained from the language $\mathcal{L}$ by adding "$\bigcirc\phi$" to the set of well-formed formulas. An ideal history $h^\star \in Ought_{(m)}$ represents the future course of an action that is ideal with respect to moment m, in accordance with the intuition that an ideal is something that ought to be the case.[6]

Definition 2.3. *Truth of $\bigcirc\phi$ in $\mathcal{M}$ at m/h is defined as follows:*
$\mathcal{M}, m/h \models \bigcirc\phi$ iff $\mathcal{M}, m/h^\star \models \phi$ for each $h^\star \in Ought_{(m)}$.

Thus the formula

$$[\alpha\ astit\colon\ \bigcirc\,[\beta\ dstit\colon \phi]], \tag{5}$$

that in contrast to formula 2 bears no contradiction, can admittedly express what happens on the deontic level of an agent's interaction when α utters an imperative sentence, and β receives what was uttered and finds herself in a situation when it is obligatory to choose between some set of alternatives.

Proposition 2.4. *Formula 5 is satisfiable.*

Proof. Suppose m/h is a moment/history index in a model

$$\mathcal{M} = \langle \mathbf{T}, \leq \Gamma, Choice, Ought\rangle.$$

Then the formula $[\alpha\ astit\colon\ \bigcirc\,[\beta\ dstit\colon \phi]]$ is satisfiable in $\mathcal{M}$ at m/h iff there is a moment $w < m$ such that:

1. $\mathcal{M}, m_1/h_1 \models \bigcirc[\beta\ dstit\colon \phi]$ for every moment $m_1 \equiv^\alpha_w m$ and for every history $h_1 \in H_{(m_1)}$, and
2. $\mathcal{M}, m_2/h_2 \not\models \bigcirc[\beta\ dstit\colon \phi]$ for some moment $m_2 \in i_{(m)}$ such that $w < m_2$ and for *some* $h_2 \in H_{(m_2)}$.[7]

[6] For more details see Chapter 3 in (Horty 2001).

[7] This particular quantification deserves special attention. On the basis of definition 1.3 "some" is expected here, but when the subformula of the formula that occurs in the "negative" choice-cell contains ought-operator something changes, as it is not possible for formulas of kind "$\bigcirc\phi$" to satisfy at some history h and do not satisfy at some history h', where h and h' belong to the same moment m. In other words formulas like "$\bigcirc\phi$" are either settled true or settled false at some moment. That is why in the situations like in point 2 of the given proof we may intrepidly switch from "some" to "every".

Using clause **1** we get:

3. $\mathcal{M}, m_1/h_1 \models \bigcirc[\beta\ dstit\colon \phi]$ for every $h_1 \in H_{(m_1)}$ iff $\mathcal{M}, m_1/h_1^\star \models [\beta\ dstit\colon \phi]$ for every $h_1^\star \in Ought_{(m_1)}$.

4. $\mathcal{M}, m_1/h_1^\star \models [\beta\ dstit\colon \phi]$ for every $h_1^\star \in Ought_{(m_1)}$ iff $\mathcal{M}, m_1/h_3 \models \phi$ for every $h_3 \in Choice^\beta_{m_1}(h_1^\star)$, and $\mathcal{M}, m_1/h_4 \not\models \phi$ for some $h_4 \in H_{(m_1)}$.

   Using clause **2** we get:

5. $\mathcal{M}, m_2/h_2 \not\models \bigcirc[\beta\ dstit\colon \phi]$ for some $h_2 \in H_{(m_2)}$ iff $\mathcal{M}, m_2/h_2^\star \not\models [\beta\ dstit\colon \phi]$, for some $h_2^\star \in Ought_{(m_2)}$.

6. $\mathcal{M}, m_2/h_2^\star \not\models [\beta\ dstit\colon \phi]$, for some $h_2^\star \in Ought_{(m_2)}$ iff either $\mathcal{M}, m_2/h_5 \not\models \phi$ for some $h_5 \in Choice^\beta_{m_2}(h_2^\star)$, or $\mathcal{M}, m_2/h_6 \models \phi$ for every $h_6 \in H_{(m_2)}$.

Since there is no contradiction within the set of clauses 1-6 this completes the proof of Proposition 2.4. □

Observe that agent β bears no possibility to escape from being obligated to seeing to it that ϕ if agent α performs an imperative and causes the deontic situation that is captured with the help of formula 5. But nevertheless agent β can still escape from seeing to it that ϕ: to the first approximation β can choose some $h_3' \notin Choice^\beta_{m_1}(h_1^\star)$ such that $\mathcal{M}, m_1/h_3' \models \neg\phi$. Naturally, formula $\neg\phi$ being true at m_1/h_3' does not make $\bigcirc[\beta\ dstit\colon \phi]$ false either at index m_1/h_1, or at any other moment choice-equivalent to moment m with respect to α's choice she does at moment w.

This intuition can be expressed in a stricter way since we have a formula of the kind

$$[\beta\ dstit\colon \neg[\beta\ dstit\colon \phi]] \tag{6}$$

at our disposal that represents the fact of β's refraining from seeing to it that ϕ.[8] We argue that the fact of issuing an imperative (namely a command) that creates the deontic situation which can be captured with formula 5, is compatible with the fact of refraining from doing what was commanded. Take notice that the same is not correct for the pair of formulas 2 and 6.

Proposition 2.5. *The conjunction of formulas*

$$[\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi]] \textit{ and } [\beta\ dstit\colon \neg[\beta\ dstit\colon \phi]]$$

is satisfiable.

Proof. Suppose again that m/h is a moment/history index in a model $\mathcal{M} = \langle \mathbf{T}, \leq, \Gamma, Choice, Ought\rangle$. Then $[\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi]]$ and $[\beta\ dstit\colon \neg[\beta\ dstit\colon \phi]]$ are consistent within $\mathcal{M}$. The calculation for the former formula in the given assertion goes just in the same way as in the proof of Proposition 2.4, so that we have the following three preconditions 1*–3*:

[8]For the reference on the refraining formula that expresses agentive not seeing to it that ϕ, see Section 4.3 in (Horty, Belnap 1995).

1.* $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi]]$;

2.* $\mathcal{M}, m \models \bigcirc[\beta\ dstit\colon \phi]$;[9]

3.* $\mathcal{M}, m/h^\star \models [\beta\ dstit\colon \phi]$, for $h^\star \in Ought_{(m)}$.

Suppose that $[\beta\ dstit : \neg[\beta\ dstit\colon \phi]]$ holds in $\mathcal{M}$ at m/h. Then, in accordance with the truth definition for *dstit*-formulas, we have the following facts:

4. $\mathcal{M}, m/h' \models \neg[\beta\ dstit\colon \phi]$ for every $h' \in Choice^\beta_m(h)$;
5. $\mathcal{M}, m/h'' \models [\beta\ dstit\colon \phi]$ for some $h'' \in H_{(m)}$.

It can be the case that histories $h^\star$ and h belong to different choice-cells, which makes formulas $[\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]]$ and $[\beta\ dstit\colon \neg[\beta\ dstit\colon \phi]]$ consistent within the model $\mathcal{M}$. □

Here is the justification of the fact of using *astit*-operator as the major device for capturing imperatives instead of *cstit* or *dstit* that are used most frequently. We proceed with two arguments. First of all the usage of the formula

$$[\alpha\ dstit\colon \bigcirc [\beta\ dstit\colon \phi]] \tag{7}$$

is simply impossible because of the contradiction it reveals.

Proposition 2.6. *Formula 7 is not satisfiable.*

Proof. Assume the opposite: then there is some model $\mathcal{M}$ and index m/h such that: $\mathcal{M}, m/h \models [\alpha\ dstit\colon \bigcirc [\beta\ dstit\colon \phi]]$. It is the case when the positive and the negative conditions for the initial formula are fulfilled. Then we have:

1. $\mathcal{M}, m/h' \models \bigcirc[\beta\ dstit\colon \phi]$ for every history $h' \in Choice^\alpha_m(h)$;
2. $\mathcal{M}, m/h'' \models \neg \bigcirc [\beta\ dstit\colon \phi]$ for some history $h'' \in H_{(m)}$.

From the first clause we can derive that $\mathcal{M}, m/h^\star \models [\beta\ dstit\colon \phi]$ for every $h^\star \in Ought_{(m)}$; from the second clause we can derive that $\mathcal{M}, m/h^\star \models \neg[\beta\ dstit\colon \phi]$ for some history $h^\star \in Ought_{(m)}$ and get the contradiction.

Secondly, using *astit* seems to be natural with respect to the interaction of a pair of agents: each of them needs a moment, where she makes choice, and the basic framework for *astit* grants at our disposal moments w and $m_1, \ldots, m_n$, where w is a choice-moment for one agent and $m_1, \ldots, m_n$ are choice-moments for the other agent.[10] In contrast to this engaging feature of *astit*, *cstit* works with one single

[9]Since clause 1* holds in particular if $\bigcirc[\beta\ dstit\colon \phi]$ holds in $\mathcal{M}$ at m_1/h_1 for every $m_1 \equiv^\alpha_w m$ and for every $h_1 \in H_{(m_1)}$, we have $\bigcirc[\beta\ dstit\colon \phi]$ settled true at every $m_1 \equiv^\alpha_w m$. As the choice-equivalence relation is symmetric, we can state that $\bigcirc[\beta\ dstit\colon \phi]$ is settled true at m also—this is the content of clause 2*.

[10]In Propositions 2.1, 2.4, 2.5 and 2.6 we implicitly consider the moment w to be the choice-moment only for the agent α, and moments $m_1, \ldots, m_n$ to be choice-moments only for the agent β. We will be consistent with this constraint in the subsequent text.

choice-moment, and in order to get another one for the second agent we need to make some transformations of the given framework for *cstit*, which finally gives us almost the same framework (with the exception of the "negative" choice-cell and "negative" histories) as we already have for *astit*. □

3 Validities and invalidities

Proposition 2.5 suggests an idea of the following formula

$$[\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi]] \to \Diamond[\beta\ dstit\colon \neg[\beta\ dstit\colon \phi]], \tag{8}$$

which says that whenever agent α issues a command that changes the deontic situation for β in such a way that β now is obligated to see to it that ϕ, β then has an ability to refrain from seeing to it that ϕ. This ability is pictured with the help of $\Diamond[\beta\ dstit\colon \neg[\beta\ dstit\colon \phi]]$ in accordance with what is proposed in (Horty, Belnap 1995: 606).

Theorem 3.1. *Formula 8 is valid.*

Proof. Assume that formula 8 is not valid. Then $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]] \wedge \neg\Diamond[\beta\ dstit\colon \neg[\beta\ dstit\colon \phi]]$ for some model $\mathcal{M}$ and some index m/h.

Then:

1. $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi]]$ and

2. $\mathcal{M}, m/h \models \neg\Diamond[\beta\ dstit\colon \neg[\beta\ dstit\colon \phi]]$.

Again the positive truth condition for the formula from clause **1** says that $\bigcirc[\beta\ dstit\colon \phi]$ is settled true in $\mathcal{M}$ at m; which means that there must be an ideal history $h^\star \in Ought_{(m)}$ such that $[\beta\ dstit\colon \phi]$ satisfies in $\mathcal{M}$ at $m/h^\star$. Note that there must be a history h' that makes the negative condition for $[\beta\ dstit\colon \phi]$ holds: $\mathcal{M}, m/h' \models \neg\phi$ for some $h' \in H_{(m)}$.

Clause **2** says that it is not possible for agent β to refrain from seeing to it that ϕ, and so we can state that in *every* history that passes through m formula $\neg[\beta\ dstit\colon \neg[\beta\ dstit\colon \phi]]$ must hold. In particular it must hold in h':

$\mathcal{M}, m/h' \models \neg[\beta\ dstit\colon \neg[\beta\ dstit\colon \phi]]$ iff either $\mathcal{M}, m/h'' \models [\beta\ dstit\colon \phi]$ for some $h'' \in Choice^{\beta}_{m}(h')$ (i), or $\mathcal{M}, m/h''' \models \neg[\beta\ dstit\colon \phi]$ for every $h''' \in H_{(m)}$ (ii).

Since histories h' and h'' are in the same choice-cell, the alternative (i) contradicts the fact that $\mathcal{M}, m/h' \models \neg\phi$ for some $h' \in H_{(m)}$.

The alternative (ii) contradicts the statement that $\bigcirc[\beta\ dstit\colon \phi]$ is settled true at m: If we accept it, we get no possibility to state that there is an ideal history $h^\star$, where $[\beta\ dstit\colon \phi]$ holds. □

Formula 5 reveals fairly unexpected results when it is substituted into the theorem of the standard deontic logic $\bigcirc(\phi \wedge \psi) \leftrightarrow (\bigcirc\phi \wedge \bigcirc\psi)$. When we substitute the impersonal ought-to-be operator "$\bigcirc$" with personal ought-to-do operator "$\bigcirc[\beta\ dstit\colon \phi]$" connected with the *astit*, it might be the case that

$$[\alpha\ astit\colon \bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])] \leftrightarrow [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]] \wedge [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]]. \quad (9)$$

However in *stit*-theory, as it is shown in (Belnap 1991: 795), it is not valid that

$$[\alpha\ astit\colon \phi \wedge \psi] \rightarrow [\alpha\ astit\colon \phi] \wedge [\alpha\ astit\colon \psi],$$

and because of that we can presumably state that while formula:

$$[\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]] \wedge [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \psi]] \rightarrow [\alpha\ astit\colon \bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])] \quad (10)$$

might be valid or at least satisfying, formula

$$[\alpha\ astit\colon \bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])] \rightarrow [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]] \wedge [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \psi]] \quad (11)$$

most likely is not valid.

Theorem 3.2. *Formula 10 is valid.*

Proof. 1. Assume that formula 10 is not satisfied at some index m/h in some model $\mathcal{M}$. Then $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]] \wedge [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \psi]] \wedge \neg[\alpha\ astit\colon \bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])]$.

Trivially by definition 1.3 we get points 2 and 3:

2. $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]] \wedge [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \psi]]$ iff $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]]$ and $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \psi]]$

3. $\mathcal{M}, m/h \models \neg[\alpha\ astit\colon \bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])]$ iff either $\mathcal{M}, m_1/h_1 \models \neg\bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])$ for some $m_1 \equiv^{\alpha}_{w} m$ and for some $h_1 \in H_{(m_1)}$ (the violation of the positive condition); or $\mathcal{M}, m_2/h_2 \models \bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])$ for every $m_2 \in i_{(m)}$ and for every $h_2 \in H_{(m_2)}$ (the violation of the negative condition).

   From point 2 we infer:

4. $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]]$ iff $\mathcal{M}, m_1/h_1 \models \bigcirc[\beta\ dstit\colon \phi]$ for every $m_1 \equiv^{\alpha}_{w} m$ and for every $h_1 \in H_{(m_1)}$ (the positive condition) and $\mathcal{M}, m_2/h_2 \models \neg\bigcirc[\beta\ dstit\colon \phi]$ for some $m_2 \in i_{(m)}$ and for some $h_2 \in H_{(m_2)}$ (the negative condition).

   Note that what is stated in point 4 with respect to formula ϕ is true also regarding ψ; so we get:

5. $\mathcal{M}, m_1/h_1 \models \bigcirc[\beta\ dstit\colon \psi]$ for every $m_1 \equiv^{\alpha}_{w} m$ and for every $h_1 \in H_{(m_1)}$ and $\mathcal{M}, m_2/h_2 \models \neg\bigcirc[\beta\ dstit\colon \psi]$ for some $m_2 \in i_{(m)}$ and for some $h_2 \in H_{(m_2)}$.

   The violation of the positive condition in point 3 implies:

6. $\mathcal{M}, m_1/h_1^\star \models \neg([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])$ for some $h_1^\star \in Ought_{(m_1)}$, which in its turn implies:

7. $\mathcal{M}, m_1/h_1^\star \models \neg[\beta\ dstit\colon \phi]$ for some $h_1^\star \in Ought_{(m_1)}$ or $\mathcal{M}, m_1/h_1^\star \models \neg[\beta\ dstit\colon \psi]$ for some $h_1^\star \in Ought_{(m_1)}$.

   The first clause of point 7 contradicts the positive condition of point 4; the second clause of point 7 contradicts the positive condition of point 5.

   The violation of the negative condition in point 3 contradicts the negative condition from point 4, or the negative condition from point 5. □

Proposition 3.3. *Formula 11 is not valid.*

Proof. Take an arbitrary model $\mathcal{M}$ and an index m/h such that:

1. $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])] \wedge \neg[\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]]$. What is said is true iff: $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])]$ and $\mathcal{M}, m/h \models \neg[\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]]$.

   The first conjunct from point 1 implies points 2 and 3.

2. $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])]$ iff $\mathcal{M}, m_1/h_1 \models \bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])$ for every $m_1 \equiv_w^\alpha m$ and for every $h_1 \in H_{(m_1)}$ (the positive condition); $\mathcal{M}, m_2/h_2 \models \neg\bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])$ for some $m_2 \in i_{(m)}$ and some $h_2 \in H_{(m_2)}$ (the negative condition).

3. $\mathcal{M}, m_1/h_1 \models \bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])$ for every $m_1 \equiv_w^\alpha m$ and for every $h_1 \in H_{(m_1)}$ iff $\mathcal{M}, m_1/h_1^\star \models [\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi]$ for every $h_1^\star \in Ought_{(m_1)}$.

   The second conjunct from point 1 implies:

4. $\mathcal{M}, m/h \models \neg[\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]]$ iff either $\mathcal{M}, m_1/h_1 \models \neg\bigcirc[\beta\ dstit\colon \phi]$ for some $m_1 \equiv_w^\alpha m$ and some $h_1 \in H_{(m_1)}$ (the violation of the positive condition) or $\mathcal{M}, m_2/h_2 \models \bigcirc[\beta\ dstit\colon \phi]$ for every $m_2 \in i_{(m)}$ and every $h_2 \in H_{(m_2)}$ (the violation of the negative condition).

Assume that the negative condition for the second conjunct from point 1 is violated. Then, as it is stated in point 4, $\bigcirc[\beta\ dstit\colon \phi]$ is settled true at every moment of a model $\mathcal{M}$, what gives no contradiction neither the positive condition, nor the negative condition for the first conjunct from point 1. That makes a model $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])] \wedge \neg[\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]]$ consistent. Since it serves as a counter-model for formula 11, we state that formula 11 is not valid. □

It is quite obvious that the lack of the parallelism between SDL and the given fragment of *stit*-theory that focuses on the deontic effects of imperatives, turns out to be a nice ground for rejecting some paradoxes of deontic logic, like McLaughlin's paradox or Ross's paradox.

Previously we stated that expression 5 pictures an imperative sentence. Now by hypothesis we state that a conjunction of two imperatives can be captured in $\mathcal{L}$ either with $[\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi \wedge \psi]]$, or with $[\alpha\ astit\colon \bigcirc ([\beta\ dstit\colon \phi] \wedge [\beta\ dstit\colon \psi])$. However that may be one can mention that the complement of the first formula in the pair can be derived from the complement of the second formula in accordance with Principe C (Chellas 1992: 502). Thus we can conclude that if formula 11 is not valid, and if $[\alpha\ astit\colon \bigcirc([\beta\ dstit\colon \phi]\wedge[\beta\ dstit\colon \psi])$ implies $[\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi\wedge\psi]]$, then formula

$$[\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi \wedge \psi]] \rightarrow [\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi]] \wedge [\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \psi]] \quad (12)$$

is not valid also.

Proposition 3.4. *Formula 12 is not valid.*

Proof. On the basis of proof 3.3 and Principle C. □

In particular the invalidity of formulas 11 and 12 makes it clear why one cannot proceed from the command to see to it that $\phi\wedge\psi$ to the command to see to it that ϕ. For example, when a child receives from her parent the command "You ought to go to the hospital and visit your grandmother", she by no means receives the command "You ought to go to the hospital and you ought to visit your grandmother", and thus she cannot eliminate the conjunction and finally get the commands "You ought to go to the hospital" or "You ought to visit your grandmother".

4 Indexed ought-sets for imperatives

The idea of indexed ought sets for *stit*-formulas comes from (Horty, Belnap 1995). The suggestion is grounded on an observation that ideal histories usually differ with respect to different agents, and so something which is ought to be for one agent may be not within things obligated for the other. In general outline, to show that agents α and β have *Ought*-sets that vary from each other, Belnap and Horty introduce indexed ought sets $Ought_{\alpha(m)}, Ought_{\beta(m)}, \ldots$ and extend the language with indexed operators $\bigcirc_\alpha$, $\bigcirc_\beta \ldots$

Then two kinds of restriction are imposed on *stit*-formulas with indexed ought-operators: The first one says that these operators can be applied only to formulas representing actions;[11] the second forbids formulas like $\bigcirc_\alpha[\beta\ dstit\colon \phi]$, where a *stit*-formula for one agent is found within the scope of the ought-operator of the other agent. The result of the second constraint is tedious: Indexed and non-indexed *stit*-formulas represent the same class of valid formulas, as it is emphasized in (Horty, Belnap 1995: 632).

[11]Strictly speaking, the same is true for non-indexed *stit*-formulas with the ought-operator too. For example, formula $[\alpha\ dstit\colon \bigcirc \phi]$ turns out to be self-contradictory: It is not possible for its subformula $\bigcirc\phi$ to be satisfied at some index m/h and not to be satisfied at some index m/h', as it is required by the negative condition for the whole formula.

Nevertheless we expect to find some use of indexed *Ought*-sets if we abandon the second constraint applied to *Ought*-set and thus get the possibility to use such formulas as

$$\bigcirc_\alpha [\beta \ dstit\colon \phi] \tag{13}$$

on the basis of the following definition:

Definition 4.1. $\mathcal{M}, m/h \models \bigcirc_\alpha[\beta \ dstit\colon \phi]$ *iff* $\mathcal{M}, m/h^{\star\alpha} \models [\beta \ dstit\colon \phi]$*, for every history* $h^{\star\alpha} \in Ought_{\alpha(m)}$*, where* $Ought_{\alpha(m)}$ *is the set of ideal histories with relation to the agent* α *that go through the moment* m*.*

Possible readings for formula 13 are: "It is obligatory for β in the face of α to see to it that ϕ"; or "With relation to α it is obligatory for β to see to it that ϕ. A command then can be captured with the help of the following formula:

$$[\alpha \ astit\colon \ \bigcirc_\alpha [\beta \ dstit\colon \phi]]. \tag{14}$$

4.1 The use of indexed ought-sets: commands

Earlier we demonstrated the favourable lack of parallelism between the SDL and elaborated *stit*-logic for imperatives that shows itself in formula 11 which is not valid in accordance with the semantics for *astit*, *dstit* and $\bigcirc$. In some instances when we use non-indexed *Ought*-sets the parallelism continues holding, which may drive us to some unwanted implications.

Consider the SDL theorem $\bigcirc(\bigcirc\phi \to \phi)$ and its *stit*-analog

$$\bigcirc (\bigcirc[\alpha \ dstit\colon \phi] \to [\alpha \ dstit\colon \phi]), \tag{15}$$

which says that "it ought to be that an agent does what he ought to do" (Horty, Belnap 1995: 624). Similary to what is said in formula 15, it might be the case that it ought to be that an agent does what she ought to do after receiving a command to do such and such. Thus it might be the case that formula

$$\bigcirc ([\alpha \ astit\colon \ \bigcirc [\beta \ dstit\colon \phi]] \to [\beta \ dstit\colon \phi]) \tag{16}$$

is unpleasantly valid.

Theorem 4.2. *Formula 16 is valid.*

Proof. Assume that it is not the case: $\mathcal{M}, m/h \not\models \bigcirc([\alpha \ astit\colon \ \bigcirc [\beta \ dstit\colon \phi]] \to [\beta \ dstit\colon \phi])$. Then there is a history $h^\star \in Ought_{(m)}$ such that $m/h^\star \models \neg([\alpha \ astit\colon \bigcirc [\beta \ dstit\colon \phi]] \to [\beta \ dstit\colon \phi])$. This is the case iff $m/h^\star \models [\alpha \ astit\colon \ \bigcirc [\beta \ dstit\colon \phi]]$ and $m/h^\star \models \neg[\beta \ dstit\colon \phi]$. $m/h^\star \models [\alpha \ astit\colon \ \bigcirc [\beta \ dstit\colon \phi]]$ iff there is a moment $w < m$ such that for every moment $m_1 \equiv^\alpha_w m$ and for every history $h_1 \in H_{(m_1)}$ $m_1/h_1 \models \bigcirc[\beta \ dstit\colon \phi]$ (the positive condition) and for some moment $m_2 \in i_{(m)}$ and some history $h_2 \in H_{(m_2)}$ $m_2/h_2 \models \neg \bigcirc [\beta \ dstit\colon \phi]$ (the negative condition). The positive condition says that $\bigcirc[\beta \ dstit\colon \phi]$ is settled true at every moment $m_1 \equiv^\alpha_w m$ and at every history $h_1 \in H_{(m_1)}$, which means that $\bigcirc[\beta \ dstit\colon \phi]$ must be true at index $m/h^\star$ since h_1 and $h^\star$ belongs to the same choice-cell $Choice^{\alpha}_{w}(h^\star)$. But at this index we also have formula $\neg[\beta\ dstit\colon \phi]$ that contradicts the fact that $m/h^\star \models \bigcirc[\beta\ dstit\colon \phi]$. This refutes our assumption and proves that formula 16 is really valid.[12] □

Indexed ought-sets can help in escaping the validity of this kind and show that it is not ought to be that agent β does some ϕ, that is ought to be done with relation to agent α, whenever α performs an imperative action (a command) and by means of this performance creates an obligation for agent β with relation to herself to do ϕ. Putting this thing formally, we argue that regardless of the fact that formula 16 is valid, formula

$$\bigcirc\left([\alpha\ astit\colon\ \bigcirc_{\alpha}\left[\beta\ dstit\colon \phi\right]] \to [\beta\ dstit\colon \phi]\right) \tag{17}$$

is not.

Proposition 4.3. *Formula 17 is not valid.*

Proof. Assume that it is the case. Then we get $m/h^\star \models [\alpha\ astit\colon\ \bigcirc_{\alpha}\left[\beta\ dstit\colon \phi\right]]$ and $m/h^\star \models \neg[\beta\ dstit\colon \phi]$. Then, exactly as in the previous argument, we have $m/h^\star \models \bigcirc_{\alpha}[\beta\ dstit\colon \phi]$ for every $h^\star \in Ought_{(m)}$. $m/h^\star \models \bigcirc_{\alpha}[\beta\ dstit\colon \phi]$ iff $m/h^{\star\alpha} \models [\beta\ dstit\colon \phi]$ for every $h^{\star\alpha} \in Ought_{\alpha(m)}$. Note that those ideal histories $h^\star$ and $h^{\star\alpha}$ may not coincide with each other and if it is the case no contradiction occurs between formulas $\bigcirc_{\alpha}[\beta\ dstit\colon \phi]$ and $\neg[\beta\ dstit\colon \phi]$. This makes formula $\neg\bigcirc\left([\alpha\ astit\colon\ \bigcirc_{\alpha}\left[\beta\ dstit\colon \phi\right]] \to [\beta\ dstit\colon \phi]\right)$ satisfactory and thus formula 17 is not valid. □

Another interesting point related to the indexed ought-sets reveals itself in the situation when agents α and β produce the same imperatives (commands) that are addressed to each other. Suppose that agent α says to agent β: "See to it that ϕ!", and suppose that α receives from β the *same* command, which means that α and β want from each other to get similar outcomes and to make ϕ true in the *same* index.[13] It might be said that the formula that depicts such situation bears a contradiction: Either because as a rule we implicitly keep in mind that α holds some authoritative position in comparison with β, and so β cannot issue a command to α; or because it is with a high probability ineffective to issue such a "symmetrical" command, as it cannot be fulfilled by both agents at the same time. Nevertheless it can be stated that formula

$$[\alpha\ astit\colon\ \bigcirc_{\alpha}\left[\beta\ dstit\colon \phi\right]] \wedge [\beta\ astit\colon\ \bigcirc_{\beta}\left[\alpha\ dstit\colon \phi\right] \tag{18}$$

is satisfiable.

[12] Although formula 16 is valid, formula $[\alpha\ astit\colon\ \bigcirc\left[\beta\ dstit\colon \phi\right]] \to [\beta\ dstit\colon \phi]$ is not. And it is not possible to infer it from premises $[\alpha\ astit\colon\ \bigcirc\left[\beta\ dstit\colon \phi\right]] \to \bigcirc[\beta\ dstit\colon \phi]$ and $\bigcirc[\beta\ dstit\colon \phi] \to [\beta\ dstit\colon \phi]$, as the last premise is not valid. Note that the situation is similar to those in SDL, where $\bigcirc(\bigcirc\phi \to \phi)$ is valid but $\bigcirc\phi \to \phi$ is not valid.

[13] If we ignore the fact that "symmetrical" commands are wanted to be fulfilled by their authors in the same index we get no conflict or contradiction between them: For example, α and β can issue the command "Open the window!" addressed to each other without worrying about possible contradiction if α wants β to fulfil the command on Monday, and β wants α to see to it what is commanded on Tuesday.

Proposition 4.4. *Formula 18 is satisfiable.*

Proof. Indeed it can be the case that

$$m/h \models \bigcirc_\alpha[\beta\ dstit\colon \phi] \text{ and } m/h \models \bigcirc_\beta[\alpha\ dstit\colon \phi].$$

Then it can be that $m/h^{\star_\alpha} \models [\beta\ dstit\colon \phi]$ and $m/h^{\star_\beta} \models [\alpha\ dstit\colon \phi]$, where $h^{\star_\alpha} \in Ought_{\alpha(m)}$ and $h^{\star_\beta} \in Ought_{\beta(m)}$. Seeing to it that ϕ then can be conceived as the joint action of α and β preceded with a reciprocal imperative. □

In order to step aside from such unnatural interpretation we put an idea that formula 13 might not grasp some essential feature of the deontic effect of issuing commands, which gives the possibility for formula 18 to be held. Here is a proposal that serves to combat against the above situation with "symmetrical commands".

Let the deontic situation for α and β that takes place after a command is issued be depicted with formula

$$[\alpha\ astit\colon (\bigcirc_\alpha[\beta\ dstit\colon \phi] \wedge \neg \bigcirc_\alpha [\alpha\ dstit\colon \phi])], \tag{19}$$

which says that when α issues a command she wishes β to fulfil what is commanded, and at the same time she is not obligated to fulfil it by herself. If formula 19 depicts commands properly (or at least in a more convenient way than formula 13 does), then we can modify formula 18 in the following way:

$$[\alpha\ astit\colon (\bigcirc_\alpha[\beta\ dstit\colon \phi] \wedge \neg \bigcirc_\alpha [\alpha\ dstit\colon \phi])] \wedge \\ [\beta\ astit\colon (\bigcirc_\beta[\alpha\ dstit\colon \phi] \wedge \neg \bigcirc_\beta [\beta\ dstit\colon \phi])]. \tag{20}$$

Proposition 4.5. *Formula 20 is not satisfiable.*

Proof. Assume that $\mathcal{M}, m/h \models [\alpha\ astit\colon (\bigcirc_\alpha[\beta\ dstit\colon \phi] \wedge \neg \bigcirc_\alpha [\alpha\ dstit\colon \phi])]$ and $\mathcal{M}, m/h \models [\beta\ astit\colon (\bigcirc_\beta[\alpha\ dstit\colon \phi] \wedge \neg \bigcirc_\beta [\beta\ dstit\colon \phi])]$. The positive condition for these *astit*-formulas provides us with the following clauses:

1. $\mathcal{M}, m_1 \models (\bigcirc_\alpha[\beta\ dstit\colon \phi] \wedge \neg \bigcirc_\alpha [\alpha\ dstit\colon \phi])$ for every $m_1 \equiv^\alpha_w m$;
2. $\mathcal{M}, m_1 \models (\bigcirc_\beta[\alpha\ dstit\colon \phi] \wedge \neg \bigcirc_\beta [\beta\ dstit\colon \phi])$ for every $m_1 \equiv^\beta_w m$.

These formulas settled true give the following clauses:

$$\begin{aligned}
&\mathcal{M}, m_1/h_1 \models \bigcirc_\alpha[\beta\ dstit\colon \phi] \\
&\qquad \text{iff } \mathcal{M}, m_1/h_1^{\star_\alpha} \models [\beta\ dstit\colon \phi] \text{ for every } h_1^{\star_\alpha} \in Ought_{\alpha(m_1)} \\
&\mathcal{M}, m_1/h_1 \models \bigcirc_\beta[\alpha\ dstit\colon \phi] \\
&\qquad \text{iff } \mathcal{M}, m_1/h_1^{\star_\beta} \models [\alpha\ dstit\colon \phi] \text{ for every } h_1^{\star_\beta} \in Ought_{\beta(m_1)} \\
&\mathcal{M}, m_1/h_1 \models \neg \bigcirc_\alpha [\alpha\ dstit\colon \phi] \\
&\qquad \text{iff } \mathcal{M}, m_1/h_1^{\star_\alpha} \models \neg[\alpha\ dstit\colon \phi] \text{ for some } h_1^{\star_\alpha} \in Ought_{\alpha(m_1)}
\end{aligned}$$

If histories $h_1^{\star\alpha}$ and $h_1^{\star\beta}$ (each containing the outcome for commands

$$[\alpha\ astit\colon (\bigcirc_\alpha[\beta\ dstit\colon \phi] \wedge \neg \bigcirc_\alpha [\alpha\ dstit\colon \phi])]$$

and

$$[\beta\ astit\colon (\bigcirc_\beta[\alpha\ dstit\colon \phi] \wedge \neg \bigcirc_\beta [\beta\ dstit\colon \phi])]$$

respectively) belong to different choice-cells in moment m_1, then commands issued by agents α and β are not "symmetrical"; if these histories belong to the same choice-cell, then the contradiction between formulas $[\alpha\ dstit\colon \phi]$ ($[\beta\ dstit\colon \phi]$) and $\neg[\alpha\ dstit\colon \phi]$ ($\neg[\beta\ dstit\colon \phi]$) reveals. □

4.2 Indexed ought-sets meet promises

It seems quite natural to use indexed *Ought*-sets to distinguish commands from promises. Whereas a command is depicted with formulas 5, 14, or formula 19, it might be possible to picture promise with the help of the following formula:

$$[\alpha\ astit\colon\ \bigcirc_\beta [\alpha\ dstit\colon \phi]] \tag{21}$$

what can be interpreted as an action of agent α who sees to it that she is obligated in the face of agent β to see to it that ϕ. Further we can modify formula 21 in order to express promises that an agent gives to herself: $[\alpha\ astit\colon\ \bigcirc_\alpha [\alpha\ dstit\colon \phi]]$. For example, the promise of giving up smoking issued by an agent in her own face would be a nice content for that.

One essential feature in acts of promising questions the acceptability of formula 21. The fact is that when agent α promises to do so and so in the face of agent β, she does not simply create an obligation for herself in the face of agent β; most probably agent α also creates an obligation to do so and so in her face *also*. We can put this idea as follows: a promise to somebody is always a promise to yourself. Promising in contrast to commanding doubles obligations. In order to express this idea formally we propose

$$[\alpha\ astit\colon\ \bigcirc_\beta \bigcirc_\alpha[\alpha\ dstit\colon \phi]] \tag{22}$$

with the following possible reading: "Agent α sees to it that from the point of view of agent β it is obligatory that from the point of view of agent α it is obligatory that agent α sees to it that ϕ".[14]

Conjecture 4.1. *Formula 22 is satisfiable.*

[14] Consider a conjecture: It is also possible to picture promise with a formula $[\alpha\ astit\colon (\bigcirc_\alpha[\beta\ dstit\colon \phi] \wedge \bigcirc_\alpha[\beta\ dstit\colon \phi])]$. This formula says ultimately that it is obligatory for α in the face of β to see to it that ϕ, while formula 22 says that it is obligatory for α in the face of β to have an obligation to see to it that ϕ. Note that this allows to express in $\mathcal{L}^+$ different types of promises: The first one needs to be fulfilled in a more exact way and so to say immediately, while the latter is obligatory not in so strong manner.

In addition to the conjecture 4.1, formula

$$[\alpha\ astit\colon \bigcirc_\beta \bigcirc_\alpha[\alpha\ dstit\colon \phi]] \to \Diamond[\alpha\ dstit\colon \neg[\alpha\ dstit\colon \phi]] \tag{23}$$

which expresses the fact that the promise implies the refraining ability for those agents who give it, is valid.

Theorem 4.6. *Formula 23 is valid.*

Proof. Assume that agent α who promises ϕ to agent β has no ability to refrain from seeing to it that ϕ. Then there must be a model $\mathcal{M}$ and an index m/h such that:

1. $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc_\beta\bigcirc_\alpha[\alpha\ dstit\colon \phi]]$ and $\mathcal{M}, m/h \models \neg\Diamond[\alpha\ dstit\colon \neg[\alpha\ dstit\colon \phi]]$.
2. $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc_\beta \bigcirc_\alpha[\alpha\ dstit\colon \phi]]$ iff there is a moment $w < m$ such that $\mathcal{M}, m_1 \models \bigcirc_\beta \bigcirc_\alpha [\alpha\ dstit\colon \phi]$ for every $m_1 \equiv^\alpha_w m$ (the positive condition).
3. $\mathcal{M}, m_1/h_1 \models \bigcirc_\beta \bigcirc_\alpha [\alpha\ dstit\colon \phi]$ iff $\mathcal{M}, m_1/h_1^{\star\beta} \models \bigcirc_\alpha[\alpha\ dstit\colon \phi]$ for every $h_1^{\star\beta} \in Ought_{\beta(m_1)}$.
4. $\mathcal{M}, m_1/h_1^{\star\beta} \models \bigcirc_\alpha[\alpha\ dstit\colon \phi]$ iff $\mathcal{M}, m_1/h_1^{\star\alpha} \models [\alpha\ dstit\colon \phi]$ for every $h_1^{\star\alpha} \in Ought_{\alpha(m_1)}$.
5. $\mathcal{M}, m_1/h_1^{\star\alpha} \models [\alpha\ dstit\colon \phi]$ iff $\mathcal{M}, m_1/h'_1 \models \phi$ for every history $h'_1 \in Choice^\alpha_{m_1}(h_1^{\star\alpha})$ (the positive condition) and $\mathcal{M}, m_1/h''_1 \not\models \phi$ for some history $h''_1 \in H_{m_1}$ (the negative condition).
6. $\mathcal{M}, m/h \models \neg\Diamond[\alpha\ dstit\colon \neg[\alpha\ dstit\colon \phi]]$ iff $\mathcal{M}, m/h \models \Box\neg[\alpha\ dstit\colon \neg[\alpha\ dstit\colon \phi]]$. Since $m \equiv^\alpha_w m_1$, we have $\mathcal{M}, m_1/h_1 \models \Box\neg[\alpha\ dstit\colon \neg[\alpha\ dstit\colon \phi]]$ as well.
7. $\mathcal{M}, m_1/h_1 \models \neg[\alpha\ dstit\colon \neg[\alpha\ dstit\colon \phi]]$ iff either $\mathcal{M}, m_1/h'_1 \models [\alpha\ dstit\colon \phi]$ for some $h'_1 \in Choice^\alpha_{m_1}(h_1)$ or $\mathcal{M}, m_1/h''_1 \models \neg[\alpha\ dstit\colon \phi]$ for every history $h''_1 \in H_{(m_1)}$. Note with respect to the first alternative that the formula $[\alpha\ dstit\colon \phi]$ must hold at some history in *every* choice-cell of the moment m_1 since the formula $\neg[\alpha\ dstit\colon \neg[\alpha\ dstit\colon \phi]]$ is settled true at m_1.[15]
8. Suppose the first alternative from point 7 takes place. Then there is no such history $h'''_1 \in H_{(m_1)}$ that $M, m/h'''_1 \models \neg\phi$ since every choice-cell contains history, where $[\alpha\ dstit\colon \phi]$ holds. This contradicts the negative condition of point 5.
9. The second alternative from point 7 contradicts positive condition from the point 5. □

[15]Otherwise if there is some choice-cell **K** that contains no history, where $[\alpha\ dstit\colon \phi]$ is true, we must say that in **K** we have $\neg[\alpha\ dstit\colon \phi]$ and thus, in combination with the fact that $[\alpha\ dstit\colon \phi]$ holds in some other choice-cell $\mathbf{K}'$, in **K** we have $[\alpha\ dstit\colon \neg[\alpha\ dstit\colon \phi]]$, which contradicts clause 6 of the given proof.

It is quite expected that in the case of commands the content of a command might not be obligatory from the point of view of an agent who is commanded to do so and so:

$$([\alpha\ astit\colon\ \bigcirc_{\alpha}\,[\beta\ dstit\colon \phi]) \wedge \neg \bigcirc_{\beta}\,[\beta\ dstit\colon \phi]. \tag{24}$$

In order to reflect this situation in the formal framework we must suppose that there is at least one choice-cell in an arbitrary frame $\mathcal{S}^{+}$, where agents α and β share no ideal history. Otherwise, for every choice-cell $Choice^{\beta}_{m}(h) = h^{\star\alpha} \cup h^{\star\beta}$ that belongs to the set of choice-cells with pairs of ideal histories for *both agents* α and β, formulas $[\beta\ dstit\colon \phi]$ and $\neg[\beta\ dstit\colon \phi]$ must be satisfied, which makes formula 24 contradictory.

Situations with promises are essentially different: If an agent who is obligated to do so and so does not consider herself as such, then the promise collapses in any circumstances. Formula

$$([\alpha\ astit\colon\ \bigcirc_{\beta} \bigcirc_{\alpha}[\alpha\ dstit\colon \phi]) \wedge \neg \bigcirc_{\alpha}\,[\alpha\ dstit\colon \phi] \tag{25}$$

that depicts this kind of situations on no conditions is satisfied.

Proposition 4.7. *Formula 25 is not satisfiable.*

Proof. To see it suppose that the first subformula of the *astit*-formula, formula $\bigcirc_{\beta} \bigcirc_{\alpha}\,[\alpha\ dstit\colon \phi]$, is settled true at moment m. Then at every history $h^{\star\beta} \in Ought_{\beta(m)}$ we have formula $\bigcirc_{\alpha}[\alpha\ dstit\colon \phi]$, which reveals the contradiction, as we also have formula $\neg \bigcirc_{\alpha}\,[\alpha\ dstit\colon \phi]$ settled true at moment m. Hence we conclude that while it is possible for an agent to receive a command and still consider oneself as not being obligated in her own face to see to it what is commanded, it is not possible for an agent to give a promise and at the same time not to be obligated in her own face to see to it what is promised. □

5 The revival of imperative inference puzzles

Now we have at our disposal effective tools that can help to solve some intricate puzzles which make difficulties for imperative logic to emerge. In the strict sense we had already solved one of them when we proved that the conjunction of commands "ϕ" and "ψ" does not imply command "ϕ" (see the proof of Proposition 3.3). This result opposes to what was said in the famous article by Alf Ross on imperatives (Ross 1944) (namely, in Ross's notation: $I(x \wedge y)$ implies $I(x)$) or in Daniel Vanderveken's proceeding on illocutionary logic (Vanderveken 1990) (where it is stated that $f(p \wedge q)$ "strongly entails" $f(p)$ and $f(q)$).

Another thing that bothers researchers of imperatives is the fact that on the basis of an indubitable validity "$\phi \rightarrow (\phi \vee \psi)$" it is possible to infer a command to do "$\phi \vee \psi$" from a command to do "ϕ". In 1995 Perloff proved that formula $[\alpha\ astit\colon \phi] \rightarrow [\alpha\ astit\colon \phi \vee \psi]$ is not valid. But this result, in spite of what is said by its author, has still little in common either with imperatives regarded directly, or with some of the essential effects they produce, and to a greater extend concerns agency and logic of action in general. Nevertheless we argue that there is a proof that when agent α commands agent β to see to it that ϕ, and the usual deontic situation then runs, she in no circumstances gives rise to the idea that a command to see to it that ϕ or ψ has been done. In other words, we argue that formula

$$[\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi]] \to [\alpha\ astit\colon \bigcirc ([\beta\ dstit\colon \phi] \vee [\beta\ dstit\colon \psi])] \tag{26}$$

is not valid.

Proposition 5.1. *Formula 26 is not valid.*

Proof. Take an arbitrary model $\mathcal{M}$ and an index m/h such that $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi]]$ and $\mathcal{M}, m/h \models \neg[\alpha\ astit\colon \bigcirc ([\beta\ dstit\colon \phi] \vee [\beta\ dstit\colon \psi])]$.
$\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi]]$ iff for every moment $w < m$ $\mathcal{M}, m_1/h_1 \models \bigcirc[\beta\ dstit\colon \phi]$ for every moment $m_1 \equiv^{\alpha}_{w} m$ and for every history $h_1 \in H_m$ and $\mathcal{M}, m_2/h_2 \models \neg \bigcirc [\beta\ dstit\colon \phi]$ for some moment $m_2 \in i_{(m)}$ and some history $h_2 \in H_{(m_2)}$. $\mathcal{M}, m_1/h_1 \models \bigcirc[\beta\ dstit\colon \phi]$ iff $\mathcal{M}, m_1/h_1^{\star} \models [\beta\ dstit\colon \phi]$ for every history $h_1^{\star} \in Ought_{(m_1)}$. $\mathcal{M}, m_2/h_2 \models \neg \bigcirc [\beta\ dstit\colon \phi]$ iff $\mathcal{M}, m_2/h_2^{\star} \models \neg[\beta\ dstit\colon \phi]$ for some history $h_2^{\star} \in Ought_{(m_2)}$.
Suppose that the antecedent of the given formula is false in $\mathcal{M}$ at m/h because of the violation of the negative condition. Then $\mathcal{M}, m/h \models \neg[\alpha\ astit\colon \bigcirc([\beta\ dstit\colon \phi] \vee [\beta\ dstit\colon \psi])]$ iff $\mathcal{M}, m_2/h_2 \models \bigcirc([\beta\ dstit\colon \phi] \vee [\beta\ dstit\colon \psi])$ for every moment $m_2 \in i_{(m)}$ and for every history $h_2 \in H_{(m_2)}$. $\mathcal{M}, m_2/h_2 \models \bigcirc([\beta\ dstit\colon \phi] \vee [\beta\ dstit\colon \psi])$ iff $\mathcal{M}, m_2/h_2^{\star} \models [\beta\ dstit\colon \phi] \vee [\beta\ dstit\colon \psi]$ for some history $h_2^{\star} \in Ought_{(m_2)}$.
There is no contradiction between formula $\neg[\beta\ dstit\colon \phi]$ and formula $[\beta\ dstit\colon \phi] \vee [\beta\ dstit\colon \psi]$ which can be true in the same ideal history $h_2^{\star} \in H_{(m_2)}$. Thus a countermodel for formula 26 exists. □

As we do not have the answer to the question about the best way of picturing a disjunction of commands, we state another formula that serves as a variation of Ross's paradox:

$$[\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi]] \to [\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi \vee \psi]] \tag{27}$$

Proposition 5.2. *Formula 27 is not valid.*

Proof. By analogy with proof of Proposition 5.1. □

By means of formulas 5 and 22 we can also get an answer to the question about the validity of modus ponens-like inference with respect to imperatives. One can ask about whether it is possible to conclude that a command to do "ψ" holds on the basis of a conjuncted command to do "ϕ" and to do "ϕ" if "ψ".[16] We argue that the formula

$$([\alpha\ astit\colon \bigcirc (\psi \to [\beta\ dstit\colon \phi])] \wedge [\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \psi]]) \to [\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi]] \tag{28}$$

is valid.

[16]Ross put this figure of a possible inference in imperative logic as follows: $I(x)$ and $I(x \to y)$ imply $I(y)$. See paper (Ross 1944: 41).

Theorem 5.3. *Formula 28 is valid.*

Proof. Take an arbitrary model $\mathcal{M}$ and an index m/h such that $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc(\psi \to [\beta\ dstit\colon \phi])]$, $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \psi]]$ and $\mathcal{M}, m/h \models \neg[\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]]$.

1. $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc(\psi \to [\beta\ dstit\colon \phi])]$ iff there is a moment $w < m$ such that $\mathcal{M}, m_1/h_1 \models \bigcirc(\psi \to [\beta\ dstit\colon \phi])$ for every moment $m_1 \equiv^{\alpha}_{w} m$ and for every history $h_1 \in H_{(m_1)}$ (the positive condition) and $\mathcal{M}, m_2/h_2 \models \neg\bigcirc(\psi \to [\beta\ dstit\colon \phi])$ for some moment $m_2 \in i_{(m)}$ and *every* history $h_2 \in H_{(m_2)}$ (the negative condition).

   As $\bigcirc(\psi \to [\beta\ dstit\colon \phi])$ holds at every $m_1 \equiv^{\alpha}_{w} m$ it must be the case that at every index $m_1/h^{\star}_1$ such that $h^{\star}_1 \in Ought_{(m_1)}$ there is $\psi \to [\beta\ dstit\colon \phi]$.

   As $\neg\bigcirc(\psi \to [\beta\ dstit\colon \phi])$ holds at some index m_2/h_2 then there must be then some index $m_2/h^{\star}_2$ such that $h^{\star}_2 \in Ought_{(m_2)}$, where formula ψ holds and formula $[\beta\ dstit\colon \phi]$ does not hold.

2. $\mathcal{M}, m/h \models [\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \psi]]$ iff there is a moment $w < m$ such that $\mathcal{M}, m_1/h_1 \models \bigcirc[\beta\ dstit\colon \psi]$ for every moment $m_1 \equiv^{\alpha}_{w} m$ and every history $h_1 \in H_{(m_1)}$ (the positive condition) and $\mathcal{M}, m_2/h_2 \models \neg\bigcirc[\beta\ dstit\colon \psi]$ for some moment $m_2 \in i_{(m)}$ and some history $h_2 \in H_{(m_2)}$ (the negative condition).[17]

   As $\bigcirc[\beta\ dstit\colon \psi]$ holds at every moment $m_1 \equiv^{\alpha}_{w} m$ it must be the case that at every ideal history which passes through moment m_1 there is $[\beta\ dstit\colon \psi]$. In particular, this fact holds with respect to the ideal history $h^{\star}_1 \in Ought_{(m_1)}$.

3. Using clause 1 and clause 2 we can get at every ideal history $h^{\star}_1 \in Ought_{(m_1)}$ formulas $\psi \to [\beta\ dstit\colon \phi]$ and $[\beta\ dstit\colon \psi]$.

   $m_1/h^{\star}_1 \models [\beta\ dstit\colon \psi]$ iff $m_1/h^{\star\prime}_1 \models \psi$ for every history $h^{\star\prime}_1 \in Choice^{\beta}_{m_1}(h^{\star}_1)$ and $m_1/h^{\star\prime\prime}_1 \not\models \psi$ for some history $h^{\star\prime\prime}_1 \in H_{(m_1)}$. As ψ holds at index $m_1/h^{\star\prime}_1$ it holds also at index $m_1/h^{\star}_1$, since histories $h^{\star}_1$ and $h^{\star\prime}_1$ belong to the same choice-cell $Choice^{\beta}_{m_1}(h^{\star}_1)$. Then we can conclude that $m_1/h^{\star}_1 \models \psi \to [\beta\ dstit\colon \phi] \wedge \psi$. Then in accordance with the modus ponens rule we get $\mathcal{M}, m_1/h^{\star}_1 \models [\beta\ dstit\colon \phi]$. Note that this calculation relates to every ideal history that belongs to every moment choice-equivalent for α and w to moment m.

4. $\mathcal{M}, m/h \models \neg[\alpha\ astit\colon \bigcirc[\beta\ dstit\colon \phi]]$ iff for every moment $w < m$ either $\mathcal{M}, m_1/h_1 \models \neg\bigcirc[\beta\ dstit\colon \phi]$ for some moment $m_1 \equiv^{\alpha}_{w} m$ and for *every* history $h_1 \in H_{(m_1)}$ (the violation of the positive condition) or $\mathcal{M}, m_2/h_2 \models \bigcirc[\beta\ dstit\colon \phi]$ for every moment $m_2 \in i_{(m)}$ and every history $h_2 \in H_{(m_2)}$ (the violation of the negative condition).

   If the violation of the positive condition occurs then there must be some ideal history where $\neg[\beta\ dstit\colon \phi]$ holds. This contradicts the fact that at every ideal

[17]It is not necessary that formula $\neg\bigcirc(\psi \to [\beta\ dstit\colon \phi])$ from clause 1 and formula $\neg\bigcirc[\beta\ dstit\colon \psi]$ from clause 2 are quantified in the same index m_2/h_2 but it might be so. Even if $\neg\bigcirc(\psi \to [\beta\ dstit\colon \phi])$ holds at some index m'_2/h'_2 it does not harm the proof.

history $h_1^\star \in Ought_{(m_1)}$ and at every $m_1 \equiv_w^\alpha m$ there must be $[\beta\ dstit\colon \phi]$ stated in clause 3.

If the violation of the negative condition takes place then we must have $[\beta\ dstit\colon \phi]$ at every ideal history $h_2^\star \in H_{(m_2)}$.This contradicts the fact that that there must be some history that goes through m_2, where $\neg[\beta\ dstit\colon \phi]$ holds in accordance with the negative condition from clause 1. □

It is worth mentioning that even if in formula 28 the formula $[\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \psi]]$ is substituted with the formula $[\alpha\ astit\colon \psi]$ or formula $[\gamma\ astit\colon \psi]$, it does not violate the validity, as we still have formula "ϕ" satisfied with respect to index m_1/h_1, for every moment $m_1 \equiv_w^\alpha m$ and every history $h_1 \in H_{(m_1)}$. Note that the formula $[\alpha\ dstit\colon \psi]$ substituted as the second conjunct into the antecedent of formula 28 does not preserve its validity, as it is possible that there is an index m_1/h_1 such that $m_1/h_1 \not\models [\alpha\ dstit\colon \psi]$ while it is still the case that $m/h \models [\alpha\ dstit\colon \psi]$. (This definitely happens because of the fact that choice-moments for agent α differ concerning formulas $[\alpha\ astit\colon \bigcirc (\psi \to [\beta\ dstit\colon \phi])]$ and $[\alpha\ dstit\colon \psi]$: for the first formula this is moment w, for the second formula, moment m). That is why the "weakened" version of formula 28,

$$([\alpha\ astit\colon \bigcirc (\psi \to [\beta\ dstit\colon \phi])] \wedge [\alpha\ astit\colon \psi]) \to [\alpha\ astit\colon \bigcirc [\beta\ dstit\colon \phi]], \quad (29)$$

where ψ can be done by any agent—α, β or some other agent γ—might be accepted as well.

Conjecture 5.1. *Formula 29 is valid.*

The same modus ponens-like validity might hold in case of promises:

$$([\alpha\ astit\colon \bigcirc_\beta \bigcirc_\alpha(\psi \to [\alpha\ dstit\colon \phi]])] \wedge [\alpha\ astit\colon \bigcirc_\beta \bigcirc_\alpha[\alpha\ dstit\colon \psi]]) \to [\alpha\ astit\colon \bigcirc_\beta \bigcirc_\alpha[\alpha\ dstit\colon \phi]]. \quad (30)$$

Formula 30 says that if agent α promises in the face of agent β to see to it that ϕ on condition that ψ holds, and if α promises in the face of β to see to it that ψ, then she promises in the face of β to see to it that ϕ. On the whole, formulas 28 and 30 most probably behave identically:

Conjecture 5.2. *Formula 30 is valid.*

6 Concluding remarks

The following theorems reflect the main results of interpreting imperatives on BTS via embedded *stit*-formulas with the $\bigcirc$-operator:

1. Theorems 3.1 and 4.6 demonstrate the refraining ability for an agent in case of command and promise respectively;

2. Theorem 3.2 demonstrates the validity of transition from two different commands to the one command that associates their content under a singular imperative operator;
3. Theorem 5.3, along with conjectures 5.1 and 5.2, shows the relevance of modus ponens-like inferences with respect to commands as well as to promises;
4. Propositions 3.3 and 5.1 forbid some transitions usual for PL and SDL.

Secondly, we regard formulas 24 and 22 as an explication of the distinction between a command and a promise, and formulas 19, 22 as expressing some of their essential features.

Finally, we want to mention some curious questions that arise owing to the usage of the formulas suggested in this article. First of all, it seems quite natural to express different types of *embedded imperatives* by means of the given formulas. For instance, the situation when an agent promises to give a command might be pictured as follows:

$$[\alpha\ astit\colon\ \bigcirc_\alpha \bigcirc_\beta([\alpha\ astit\colon\ \bigcirc_\alpha [\beta\ dstit\colon \phi]])]. \tag{31}$$

Moreover, this formula might be considered as a conclusion from the premises $[\alpha\ astit\colon\ \bigcirc_\alpha (\psi \to [\beta\ dstit\colon \phi])]$ and $[\alpha\ astit\colon\ \bigcirc_\alpha \bigcirc_\beta[\alpha\ dstit\colon \psi]]$, where the first mentioned premise says that agent α commands β to see to it that ϕ on condition that ψ holds, and the second premise says that α promises β that she is going to see to it that ψ. Unfortunately, we immediately get into difficulties as soon as we try to settle the truth definition for formulas with several *astit*-operators because then an arbitrary frame contains several witness-moments, and their correlation reveals contradiction, as it is shown in (Chellas 1992: 503–505) for the formula $[\alpha\ astit\colon [\beta\ astit\colon \phi]]$.

The second question arises mainly because of the usage of indexed *Ought*-sets. Since we get an instrument that can moot the "transitive" properties of commands and promises, it might be interesting to check, for instance, the validity of the formula

$$([\alpha\ astit\colon\ \bigcirc_\alpha [\beta\ dstit\colon \phi]] \wedge [\beta\ astit\colon\ \bigcirc_\alpha [\gamma\ dstit\colon \phi]]) \to [\alpha\ astit\colon\ \bigcirc_\alpha [\gamma\ dstit\colon \phi]], \tag{32}$$

which says that the command to see to it that ϕ in the face of agent α given to agent β by agent α, and then given (or transmitted) to agent γ by β, equals command to see to it that ϕ in the face of α given to γ by α directly.

The third question touches upon the possibility of expressing *permissions* in *stit*-theory, as permissions just like obligations are deontic consequences of performed imperatives. A quick glance on the language of *stit*-theory and its semantics suggests an idea that historical possibility operator $\Diamond$ can be used in order to express permissions granted from one agent to another. For instance, the formulas $[\alpha\ astit\colon \Diamond[\beta\ dstit\colon \phi]]$ or $[\alpha\ astit\colon [\alpha\ astit\colon \Diamond[\beta\ dstit\colon \phi]]]$ might be good candidates for that.

At last we find rather appealing the idea of a "shared" ideal history mentioned at the end of subsection 4.2. Together with a certain class of frames such that at each moment there are only two choice-cells, this idea can help to express some intriguing features of an imperative agency. In particular, it is not possible for formula $[\alpha\ astit\colon \bigcirc_\alpha [\beta\ dstit\colon \phi]]$ to be satisfied on such class of frames when there is no choice-cell that associates ideal histories $h^{\star\alpha}$ and $h^{\star\beta}$. This can be interpreted as an unsuccessful command under the stipulation that β does not agree with what is commanded by α.

All these items are subject of our particular concern in the course of the future investigation of imperatives in the framework of *stit*-theory supplied with indexed *Oughts*.

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