FORMAL PHILOSOPHY 2021

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FORMAL PHILOSOPHY OF SCIENCE: STATEMENT AND SET-THEORETIC APPROACHES

Abstract. The so-called received view of scientific theory, initially proposed by logical positivists, represented scientific theory as a set of statements of some formal language, ordered by syntactic relation of deductive derivability. By late 1960s this strategy had met with severe criticism which called into question the effectiveness of formal methods in philosophy of science. The set-theoretic (semantic) approach in formal philosophy of science, which can be treated as natural development of received view, is based on the concept of model in Tarski's sense and is capable of neutralizing the most part of these objections.
Keywords: received view, set-theoretic approach, model, structure of theory, Suppes, Suppe.

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The standard (statement or syntactic) approach to the formalization and analysis of scientific theories, initially put forward by logical positivists, linked the possibility of successful resolution of certain epistemic problems (such as differentiation of analytic and synthetic statements of scientific theories, search of elementary and doubtless foundations of scientific knowledge etc.) with the idea of strict explication of these problems in some artificial language. The scientific theory which might be subject to philosophical analysis should be construed as a partially interpreted axiomatic system with its fundamental laws taken as axioms and formulated in some theoretical language L_T. Observable conclusions that could be drawn from statements of theoretical languages should be expressed in a separate observation language (language of the observables) L_O. Connection between the concepts, belonging to L_T, L_O ought to be implemented by means of so-called coordinate rules (reduction statements). Only concepts from L_O could be treated as directly meaningful.

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In addition to the above-mentioned characteristics of this approach to the analysis of scientific knowledge as statement/syntactic, H. Putnam dubbed it "the received view".

Thus, for the purpose of successful methodological analysis a certain scientific theory should be exposed as (partially) interpreted set of statements of some formal language, which is (partially) ordered by the (syntactic) relation of deductive derivability. The common definition of scientific theory "in general terms" as a set of statements, closed under relation of deductive derivability / ordered by this relation, harks back to the standard/received view of scientific theories.

It should be noted, that as usual the concept of standard treatment of scientific theories is identified with the concept of standard formalization (standard formalization of a theory is its complete formalization/axiomatization in the first order predicate logic with equality). Such identification proves to be utterly incorrect, although almost all critics of standard approach do not tell these notions apart. This fact led to serious misinterpretation of the whole program of logical positivism.

Among the objections which had been raised against received view by late 1960s, were the following ones:

Theories of empirical sciences cannot be effectively and naturally formalized in the language of first order predicate logic; but even if such formalization had been fulfilled, it would have fallen short of its main aim—precise identification/ characterization of the theory. According to the theorem of the existence of a non-standard model, if a theory has at least one infinite model, it also has an infinite model of any cardinality, i.e. via such type of formalization intended models of the theory cannot be discriminated from unintended ones and axioms of the theory will be realizable on some "paradoxical" domains.

Theories are not purely linguistic objects—sets of statements / well-formed formulas of some artificial language, linked by the syntactic relation of deductive derivability. According to the core principles of logical semantics, only statements (propositions) can be evaluated as true or false; hence, the category of truth as well as the traditional (correspondent) theory of truth prove completely inapplicable to the analysis of real structure of scientific knowledge.

As a result, formal methods of the analysis of scientific knowledge are deemed "inappropriately rigid" for the purposes of philosophy and methodology of science.

However, certain drawbacks of the statement view (being also seriously exaggerated) do not inevitably lead to the total rejection of formal methods in the philosophy of science.

One of the most natural alternatives to the statement representation of scientific theories is the set-theoretic (model-theoretic, structuralist, semantic) approach to the formalization of scientific theories, aiming at direct characterization of all intended models of the theory.

This strategy in the formal philosophy of science is based on the concept of (formal) model of a theory (in Tarski's sense), admits of evaluation of the theory as true or false in a "correspondent" sense (with certain reservations though) and is free from the most part of the drawbacks of the received view (Suppes 1961).

Scientific theories can be treated as hierarchies of models of various type-theoretic levels. A model is understood as a possible realization of a theory satisfying its axioms. Possible realization is an ordered triple of elements $\langle D, R, F \rangle$, where D is a non-empty domain, R is a non-empty set of relations (predicate symbols) defined on D, and F is a (possibly empty) set of functions defined on D.

A certain possible realization is a model of some theory iff all fundamental laws (axioms) of the theory are true under interpretation in terms of $\langle D, R, F \rangle$. The structure of a theory is the ordered set of the models of different types.

Since the structure of the theory is a set-theoretic, i.e. non-linguistic object, the concepts of truth and falsity cannot be applied to it directly. Instead the following *fundamental empirical hypothesis* of the theory should be formulated and estimated:

There is a certain (structural) correspondence between the elements of the domain of empirical theory (models of data) and highly idealized models of the fundamental laws (axioms) of the theory, which provides adequate representation of observable phenomena in abstract models of the theory. The rigour of such correspondence (isomorphism, partial isomorphism, embeddability of the models of the theory into each other) may vary in different versions of the set-theoretic approach.

Provided such correspondence is verified (for empirical sciences this procedure cannot be purely formal), the theory proves (approximately) true in correspondent sense.

According to Frederick Suppe, one of the adherents of semantic approach, there are at least two different stages in the process of "methodological ascent" from the observables to fundamental axioms of the theory:

1) transition from observable phenomena to the so-called data models (hard data), which can be also called scientific facts in a proper sense;
2) transition from data models to fundamental postulates of the theory.

Coordinate rules in their traditional interpretation used to mix up these two stages that resulted in elimination of models from theory analysis.

The first stage presupposes statistical processing of empirical data and their interpretation in terms of (at least tentatively) accepted theories—conceptualizing of the theory domain in some theoretical vocabulary and the construction of its initial model (physical system). This stage is inherently counterfactual: e.g. in classical mechanics we do not deal with "tangible" velocities, accelerations and masses, but rather with logically possible values of such parameters under some idealized conditions (frictionless movement of non-dimensional mass-points). By means of a special empirical methodology this strategy enables us to predict and explain the dynamics of some observable phenomena which do not satisfy certain idealized conditions: how the actual observable phenomena would behave were the idealized conditions met. (In this way from physical models we get causally possible ones; Suppe 1989.) As this idea was stated by

Stathis Psillos,

> A description D approximately fits a state S (i.e. D is approximately true of S) if there is another state S' such that S and S' are linked by specific conditions of approximation, and D fits S' (D is true of S').
>
> So, for instance, a theoretical law is approximately true of the world, if it is strictly true in a world which approximates ours under certain conditions... (Psillos 1999: 268)

The main function of a scientific theory is to pick out from the class of logically possible theory models the causally possible ones. A theory can be regarded as empirically true iff the class of its logically possible models coincides with the class of its causally possible models (or—not so rigorously—the class of its logically possible models "covers completely" the whole class of its causally possible models).

Empirical evaluation of the fundamental hypothesis of the theory requires a formulation and a proof of the so-called *representation theorem*, which underlies a theory of measurement (correlates quantitative numerical models of observable phenomena with its qualitative empirical "prototypes") and demonstrates the presence of common invariant characteristics, intrinsic to the models of the different types. These invariant characteristics allow for a description of a class of possible models of the theory in terms of a single set-theoretic structure.

Apart from the language of a set theory (informal or axiomatized), higher-order languages of predicate logic or language of category theory might be successfully used for the description of the theories of empirical sciences.

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