Manuel Arnulfo Cañas Muñoz[1]

THE INTERNALIST JUSTIFICATORY FUNCTION OF REDUCTIO AD ABSURDUM[2]

Abstract. In the present essay it is discussed the main features of reductio ad absurdum as a source of justification. These properties are consequences of the employment of contradictions as a reason for proving if a statement is true. Although a valid deductive argument can build an internalistic justification, I would suggest that the justification obtained by reductio ad absurdum cannot be externalist. This is because contradictions as reasons can be considered internal states from different definitions.
Keywords: reductio ad absurdum, indirect method, internalism, contradiction, epistemic justification.

Для цитирования: *Cañas Muñoz M. A.* The Internalist Justificatory Function of *Reductio ad Absurdum* // Логико-философские штудии. 2021. Т. 19, № 1. С. 105–109. DOI: 10.52119/LPHS.2021.92.99.006.

Although it could be obvious that any deductive argument and method can confer internalist justification, the main point of the present essay is to argue that demonstration through *Reductio ad Absurdum* (RA) is sufficient condition to obtain an internalistic justification. The latter is because RA requires the mental construction of impossible entities and, in some sense, being introduced to an absurd world. These requirements are not only high intellectual achievements, but they also depend in a strong way on the mental states, perspective and available items of the thinker that develops the demonstration. For that dependency, I would suggest that the use of contradiction as a justifier (with the steps involved in its employment) is sufficient to consider the resulting justification there as internalistic from many of the main kinds of contemporary internalism, namely mentalism, perspectival internalism and accesibilism.

1. Contradiction as a Source of Demonstration

The RA or proof by contradiction is an indirect method of demonstration that has two main applications: as a tool of research and as a means of exposition (Polya 1973). It is an indirect method because the statement to be proved (q) is inferred from an

[1] *Manuel Arnulfo Cañas Muñoz*, PhD student, University of Salamanca; part-time professor, National Pedagogical University, Mexico.
id00771963@usal.es

[2] I am very grateful for the support of Atocha Aliseda, Draducciones, Sandra López Velasco, and Alicia Ávila.

absurd or contradiction rather than from certain premise (p)[3]. In this way, proof by contraposition can be considered as another indirect method, because there, the major proposition is demonstrated through the true value of the equivalent contrapositive statement ($\neg q \rightarrow \neg p$). In classical mathematical logic, RA can be presented as a tautological axiom, $(A \rightarrow B) \rightarrow ((A \rightarrow \neg B) \rightarrow \neg A)$[4]; and as derived rule (Epstein, 2011):

$$\frac{(\neg A \rightarrow B) \wedge (\neg A \rightarrow \neg B)}{\therefore A}$$

In this inferential structure A represents the statement that has to be demonstrated, the two premises—($\neg A \rightarrow B$), ($\neg A \rightarrow \neg B$)—deduce a contradiction from the denial of A; A and B are metalinguistic variables that can denote any well-formed formulas, including conditionals. In the next example (Hine 2019), the conclusion A is "the difference of any rational number and any irrational number is irrational''; and the contradiction is that certain number y is rational and irrational.

1. The difference of some rational number x and some irrational number y is rational.
2. The number x is equivalent to a certain fraction (a/b) among two integers a and b, a/b.
3. x less y is equal to a certain fraction (c/d) between two integers c and d, c/d.
4. y is equivalent to the difference of the fractions (a/b) and (c/d).
5. The difference of the fractions (a/b) and (c/d) is a certain fraction (e/f) among two integers e and f.
6. y is a rational number that is equivalent to e/f.

$\therefore$ 7. The difference of any rational number and any irrational number is irrational.

2. Construction and Destruction of Impossible Entities

Polya (1973) offers an account of RA as an open-minded procedure, which explains the difficulties involved in it as an intellectual achievement. According to Polya, RA not only states the truth of a statement, but it also shows why it is true. This is because RA must be employed when some trials to make consistent certain proposition have failed; the involved contradiction explains why the opposite statement is true.

RA demands a high intellectual effort because it is easier retaining in mind truths than falsehoods. RA urges us to create something impossible and clearly remembering the elements of the resulting contradictory entities (such as the final prime number, the fraction equivalent to $\sqrt{2}$, and others); if those points are not kept through the proof, the RA fails. RA is not an obvious procedure because it is hard maintaining not

[3]Following the convention, "p" and "q" represent propositional variables.

[4]"A" and "B" are used as metalinguistic variables that mean well-formed formulas.

only something that is not true, but even the elements that compose a falsehood. This intellectual effort can arise frustration (Leron 1985), the feeling that what had been created through the demonstration was vainly retained. This is because RA requests, unlike other sorts of demonstration, not only the mental construction of an impossible entity, but even destroying this artificial picture.

Another major obstacle with proof by contradiction consists in identifying where the absurd can be found. The contradiction might directly appear with one of the original premises or may result with inferred propositions during the demonstration (Jourdan, Yevdokimov 2016). But in addition, the absurd can be found with a well-known falsehood that is not explicitly a contradiction (i.e. $1 = 0$). This is the situation with the following proof by RA of "If $a \times b = 0$, then $a = 0$ or $b = 0$", where a and b are real numbers (Antonino, Mariotti 2006).

1. The product of two integers a and b is equivalent to zero, but neither a, nor b is equivalent to zero.
2. The fraction $1/a$ is the multiplicative inverse[5] of a.
3. The fraction $1/b$ is the multiplicative inverse of b.
4. $a \times b$ multiplied by $1/a$ is equivalent to zero mulplied by $1/a$.
5. $a \times b$ multiplied by $1/a$ is equal to $b \times 1$.
6. b multiplied by $1/b$ is equivalent to zero mulplied by $1/b$.
7. b multiplied by $1/b$ is equal to 1.
8. 1 is equivalent to zero.

∴ 9. The difference of any rational number and any irrational number is irrational.

For the latter, there are at least three intellectual achievements that makes RA a complex method:

1. Remembering not only a falsehood, but even its components and consequences.
2. Mentally constructing and destroying an impossible entity.
3. Finding an absurd not only through the premises and their implications, but even with falsehoods that are known to be false beyond the demonstration.

3. RA and an Internalistic Justification

The main internalist thesis is that the justificatory status of any statement for a person supervenes on conditions that are internal to the person (Fumerton 2007). From a broad point of view, there are at least three general groups of internalist conception as distinct definitions of what is internal, namely mentalism, perspectival internalism and accesibilism.

When mentalism (Conee, Feldman 2004) defends that the justificatory status of a proposition depends on internal states, it is referring to the thinker's mental states.

[5] The product of the multiplicative inverse of the number a and a is equal to 1.

This conception is not restricted to discursive justification, mental states include not only justified beliefs or knowledge, but even sensations, thoughts, and dispositional states. As it has been mentioned, RA demands the intellectual construction of a strictly impossible entity or circumstance. If what is elaborated during a RA were not impossible it would not demonstrate anything. Can these absurd entities be located out of the mental field? I would suggest that this cannot be done without severe difficulties. It is evident that in the empirical and physical world there are not contradictory objects or affairs, because if they were impossible, they could not physically exist. But even accepting something as mind-independent abstract entities it is at least dubious that they could be plenty contradictory. Thus, since the justifier (something with a justifying role) in a RA cannot exist out of the thinker's mind, any advocate of mentalism must consider this sort of demonstration as an internalistic justification.

Perspectival internalism takes the internal grounds that justify a proposition as the justified beliefs that are inside the subjective conception of certain epistemic agent (Alston 1986). In this sense, perspectival internalism is limited to justification by reasons from the subject's perspective. In a proof by contradiction its extension and steps are not fully determined for the proper main proposition to be proved. This explains why the same statement can be established using distinct contradictions. Where the absurd must be found is an inquiry in which the beliefs of the thinker display a fundamental role, because the contradiction can be discovered through different ways. If the justification obtained by contradiction is conditioned for what the subject justifiably believes, then it can be considered as internalistic from this second conception of internalism. It is always possible that the RA could not be successful because the thinker has not had enough of his/her beliefs in consideration; a subject without any beliefs related to the main statement to be demonstrated cannot prove anything by contradiction.

The third kind of internalism mentioned, accesibilism or access internalism, identifies the internal conditions with justifiers to which a subject has certain special access. Chisholm (1966) assumes as internal the justificatory conditions that are accessible merely by reflecting upon his own conscious states without any other short of support, as direct awareness, or acquaintance (Fumerton 2007). The clearest obstacle to consider the grounds obtained through proof by contradiction as directly accessible arises when this access is understood as a non-inferential one. This is because in any RA the absurd seems to be the result of many inferences involved in its steps; in appearance the contradiction is not directly accessible. However, it can be argued that contradiction cannot be partially grasped, because it only results when its elements are taken together, if they are separated, for example in different moments, the contradiction disappears; thus, what could be taken as inferentially obtained is the components of the contradiction. The absurd in a RA when is reached is taken as a whole, the recognition of the proper contradiction is directly made without inferences behind.

4. Conclusion

By the precedent discussion, the employment of a contradiction as a method of proof implies several consequences that are not shared with other deductive procedures. These non-shared features allow the asserting that the construction of a RA is enough to obtain an internalistic justification, because an absurd can be considered an internal element in three main ways. Since contradictions neither can be physical objects or circumstances, nor can they be abstract mind-independent entities, them must be regarded as mental pictures. In addition, the discovery of a contradiction depends on the beliefs that compose the thinker's perspective. This is because in RA there is not a unique contradiction that could be located. Finally, when something is taken as an absurd in a demonstration, this is grasped as a whole; by the latter there is only one intellectual act in the recognition of something as a contradiction. Being aware of an absurd is not inferential in nature.

References

Heath 1921 — *Heath T.* A History of Greek Mathematics. Oxford University Press, 1921.

Hine 2019 — *Hine G.* Proof by Contradiction: Teaching and learning considerations in the secondary mathematics classroom // Australian Mathematics Education Journal. 2019. Vol. 1, no. 3. P. 24–28.

Jourdan, Yevdokimov 2016 — *Jourdan N., Yevdokimov O.* On the Analysis of Indirect proofs: Contradiction and Contraposition // Australian Senior Mathematics Journal. 2016. Vol. 30, no. 1. P. 55–64.

Leron 1985 — *Leron U.* A Direct Approach to Indirect Proofs // Educational Studies in Mathematics. 1985. Vol. 16. P. 321–325.

Polya 1973 — *Polya U.* How to Solve it. A new aspect of Mathematical Method. Princeton University Press, 1973.