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PENALTY LOGIC: PARACONSISTENCY AND APPLICATIONS

Abstract. Penalty logic is a non-classical non-monotonic logic which allows us to demonstrate the power of belief, the level of truth or the reliability of data that we are using. This logic can be useful to model human reasoning or to replace the penalty function in the sphere of Machine Learning. We can also talk about paraconsistency of this logic and suggest the definition of contradiction without negation.

Keywords: penalty logic, paraconsistent logic, non-monotonic logic, Machine Learning, contradiction without negation.

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Penalty logic was first introduced by Gabi Pinkas in 1995 in his article "Reasoning, non-monotonicity and learning in connectionist network that capture propositional knowledge" (Pinkas 1995). Penalty logic is a non-classical logic where each formula can represent the power of faith, belief, reliability of the source of our knowledge, etc.

The inference relation in Penalty logic is non-monotonic.

Let us point out some basic concepts of Penalty logic, given in Pinkas article.

Definition 1. A Penalty logic well formed formula (PLOFF) ψ is a finite set of pairs. Each pair is composed of a positive real number (p_i), called the penalty, and a standard propositional formula, called the assumption (ϕ): $\psi = \langle \phi_i, p_i \rangle$.

Definition 2. The violation rank of a PLOFF ψ is the function ($Vrank_\psi$) that assigns a real-valued rank to each of the truth assignments. The $Vrank_\psi$ for a truth assignment $\vec{x}$ is computed by summing the penalties for the assumptions of ψ which are violated by the assignment; i.e. $Vrank_\psi(\vec{x}) = \sum_{\vec{x} \nvDash \varphi_i} p_i$.

Definition 3. The models that minimize the $Vrank_\psi$ function are called the preferred models of ψ: $\{\vec{x} | min_{\vec{y}}\{Vrank_\psi(\vec{y})\} = Vrank_\psi(\vec{x})\}$.

Definition 4. Let φ, ψ be PLOFFs, a PLOFF ψ semantically entails φ ($\psi \vDash \varphi$) iff all the preferred models of ψ are also the preferred models of φ: $\Gamma_\psi \subseteq \Gamma_\varphi$.

Penalties can also be interpreted as different levels of truth (similar to fuzzy and probabilistic logics). In such an interpretation we say that if PLOFF consists of one ordered pair with penalty $+\infty$, it means that we are dealing with an identically true (in

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the classical way) formula which cannot be violated. Visa versa for 0, we can violate that formula as much as we want. Based on what have been said by Pinkas and on what we know about fuzzy logic, let us reinterpret the entailment.

To do that let us say how we can compute the total penalty of a PLOFF. One of the possible ways to do that is to calculate the arithmetic mean penalty of the PLOFF. For PLOFFs that contain penalty of $+\infty$ the total penalty will be equal to $+\infty$.

Definition 5. PLOFF ψ entails PLOFF φ ($\psi \vDash \varphi$) in our new interpretation iff the total penalty of φ is equal or less than the total penalty of ψ.

Definition 6. PLOFF is a contradictionary formula (or formula that contains contradiction) iff a PLOFF contains ordered pairs $\langle\varphi, +\infty\rangle$ and $\langle\varphi, 0\rangle$.

Based on our new definitions of contradiction and entailment, we can conclude that a contradictionaty PLOFF can only entail PLOFFs that include $+\infty$ penalties.

"Paraconsistent logic are those which permit inference from inconsistent information in a non-trivial fashion" (Priest 2002).

We have already stated that contradiction in Penalty Logic is a PLOFF that contains both formulas $\langle\varphi, 0\rangle$ and $\langle\varphi, +\infty\rangle$. We also found out that those PLOFFs can only entail PLOFFs that include $+\infty$ penalties. Therefore, we can conclude that Penalty Logic permits inference from inconsistent information in non-trivial fashion.

The paraconsistency of Penalty Logic makes it useful for machine learning techniques. For example, we can work with data based on an expert knowledge that often contain a contradiction, but we can be sure that we can save identically truthful information.

Penalty Logic can be used in machine learning and neural networks to replace the penalty function. The penalty function is used in computer science and especially in the area of Machine Learning. Penalty functions are mostly used to convert constrained problems into unconstrained problems by introducing an artificial penalty for violating the constraint. This process can be called "external" toward the data that we work with. On the other hand, penalty logic can be used to make this process "internal," which means that we can calculate penalties while working with our data itself. To do that we also need to introduce some operators (such as the Merge operator, which was introduced by Pinkas in his article). This is a possibility to make the computation process for Machine Learning faster.

References

Pinkas 1995 — *Pinkas G.* Reasoning, nonmonotonicity and learning in connectionist networks that capture propositional knowledge // Artificial Intelligence. 1995. Vol. 77. P. 203–245.

Priest 2002 — *Priest G.* Paraconsistent logic // Handbook of Philosophical Logic / ed. by D. M. Gabbay, F. Guenthner. Vol. 6. Springer, 2002. P. 287–290.