ЛОГИКА СЕГОДНЯ

Elena Popova[1]

MERGING EPISTEMIC AND TEMPORAL MODELS: A HISTORY-FREE APPROACH[2]

Abstract. There are two approaches to merging temporal and epistemic models. The first one consists in starting with a temporal model and enriching it with epistemic dimension (as temporal epistemic logic), while the second one is supposed to start with an epistemic model introducing temporal dimension (dynamic epistemic logic, epistemic temporal logic). The proposed evolutionary epistemic model (EEM) is based on the standard epistemic model with an evolutionary relation. EEM captures knowledge changes in terms of the evolution of worlds included in different epistemic contexts. Unlike other temporal-epistemic models, EEM is free from the concept of history and enriched with quantification operators over the worlds' evolution stages.
Keywords: epistemic logic, temporal logic, dynamic epistemic logic (DEL), epistemic temporal logic (ETL), temporal epistemic logic (TEL).

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1. The Interaction of Knowledge and Time

In terms of agency, epistemic states and temporality are closely related. Knowledge changes over time which is revealed in learning and forgetting some pieces of information. There are various ways of logical formalization of the interaction of epistemic states and temporality that are suitable only for a particular class of scenarios. One of the most important theoretical problems lies in the lack of a universal logical approach to the full range of possible changes in agents' knowledge over time.

[1] *Elena Popova*, International Laboratory for Logic, Linguistics and Formal Philosophy, National Research University Higher School of Economics.
elpopova@hse.ru

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2. On Logics for Knowledge and Time

Fusion is the easiest way to join epistemic and temporal modalities (Dixon, Fisher, Wooldridge 1998; Gabbay *et al.* 2003). This method generates a bimodal logic with two independent relations on a set of possible worlds. **Fusion of epistemic and temporal logics** generates a model with two independent modalities for knowledge and time working on the epistemic and temporal relations independent from each other. This approach allows investigating the huge variety of possible development of knowledge over time. On the other hand, the fusion of epistemic and temporal logics lacks a close connection between temporal and epistemic aspects. It fails to explicate the temporal perspective of knowledge evolution.

Dynamic epistemic logic (DEL) provides a closer connection between temporality and epistemic states (van Ditmarsch, van der Hoek, Kooi 2007; van Benthem, van Eijck, Kooi 2006). Lacking the temporal modalities, DEL captures the idea of the passage of time in the event model. Therefore, time is considered not as something external (as in fusion models) but as something embedded in the ontology and coinciding with a sequence of events. Product update in DEL machinery generates the dynamics of knowledge and explains the causes of its changes.

Nevertheless, there is no particular tool for ordering the events in DEL. Conversely, **epistemic temporal logic (ETL)** includes a protocol in its system (Parikh, Ramanujam 2003). A protocol provides a systematic description of event sequences. There are no temporal modalities in ETL (similar to DEL). The conception of time changes is captured by the operator $[e]\varphi$ which is read as "after the event e, φ will be true". Histories are considered as possible worlds. ETL models reflect both knowledge evolution and development of the world due to the protocol. On the other hand, ETL lacks the product update or any alternative mechanism explaining how epistemic states are generated by such event sequences.

There were several successful attempts to fill these gaps of both ETL and DEL by merging them. The integration of frameworks of these logics is denoted as **merging DEL and ETL** (Hoshi 2010; van Benthem *et al.* 2009). The main ideas are based on the introduction of the product update mechanism into ETL model or adding a protocol into DEL.

Merging DEL and ETL has no temporal operators for quantification over the histories. Event operators capture only the next stage of the epistemic state development, which makes it difficult to consider the entire model. **Temporal epistemic logic (TEL)** designed for model checking considers knowledge evolution in the notion of interpreted systems (Fagin *et al.*, 1995; Lomuscio, Penczek 2015). TEL is based on computation tree logic (CTL) with quantifiers over branching-time paths. It is possible to enrich merging DEL and ETL framework with such kinds of quantifiers.

We introduce the **Evolutionary Epistemic Model (EEM)**, which considers changes in agents' epistemic states as the evolution of the same worlds which occurs due to information events. We highlight the key features of EEM:

- EEM describes the evolution of the worlds with unchangeable facts, these worlds are included in various epistemic contexts;
- EEM does not require the concept of history (in contrast to merging DEL and ETL);
- EEM is enriched with quantification operators over the world's stages or paths (similarly to CTL).

3. Evolutionary Epistemic Model

EEM is an extension of the standard model of epistemic logic with a basic Kripke model $\mathcal{M} = (W, \{\sim_i\}_{i \in A}, V)$, where W is a set of possible worlds, $\sim_i$ is an epistemic relation on W for every agent i in the set of agents A and $V : Var \mapsto \mathcal{P}(W)$. The language of the epistemic logic $\mathcal{L}_{EL}$ is defined in the standard way.

Definition 1 (syntax)**.** The language of $\mathcal{L}_{EEL}$ is generated by the following grammar:

$$\varphi, \psi ::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid K_i\varphi \mid {}^{\Box}X\varphi \mid {}^{\Box}F\varphi \mid {}^{\Box}G\varphi,$$

where $p \in Var$, that is, the set of propositional variables, and $i \in A$.

Modal operators ${}^{\Box}X$, ${}^{\Box}F$, ${}^{\Box}G$ have their analogies in CTL and TEL logic. Part ${}^{\Box}$ is interpreted as "in all possible options for the development of the scenario" and parts X, F, G are interpreted as "in the next evolution stage", "sometime in the future" and "always in the future", respectively.

There are dual operators defined in the following way: ${}^{\Diamond}X\varphi \equiv \neg{}^{\Box}X\neg\varphi$, ${}^{\Diamond}F\varphi \equiv \neg{}^{\Box}F\neg\varphi$, ${}^{\Diamond}G\varphi \equiv \neg{}^{\Box}G\neg\varphi$. We use $K_i^?\varphi$ as a brief notation for $K_i\varphi \vee K_i\neg\varphi$.

EEM describes the epistemic evolution of possible worlds. We introduce an evolutionary relation $\rightsquigarrow$ that connects different stages of the same world.

Definition 2 (evolutionary epistemic model)**.** EEM is defined as follows:

$$\mathcal{M}^{\mathcal{E}\mathcal{E}} = (W, Root, \{\sim_i\}_{i \in A}, \rightsquigarrow, V),$$

where $(W, \{\sim_i\}_{i \in A}, V)$ is a standard Kripke model for epistemic logic, $Root \subseteq W$ and $Root \neq \emptyset$. For all $x \in Root$, x is a root of a tree structure in relation to $\rightsquigarrow$. $\rightsquigarrow$ is the relation of the epistemic evolution of the world on W. $x \rightsquigarrow y$ is interpreted as "y is the next stage of x world's evolution". This relation has the following properties:

· inverse functionality: $\forall x \forall y \forall z((y \rightsquigarrow x \wedge z \rightsquigarrow x) \rightarrow y = z)$

· irreflexivity: $\forall x \neg(x \rightsquigarrow x)$

· asymmetry: $\forall x \forall y(x \rightsquigarrow y \rightarrow \neg y \rightsquigarrow x)$

· intransitivity: $\forall x \forall y \forall z((x \rightsquigarrow y \wedge y \rightsquigarrow z) \rightarrow \neg x \rightsquigarrow z)$

· the constancy of the facts of the world: $\forall x \forall y((x \vDash p \wedge x \rightsquigarrow y) \rightarrow y \vDash p)$

· no random and unrelated worlds: $\forall x \exists y (x \in \{W/Root\} \to y \rightsquigarrow x)$
· synchrony: $\forall x \forall y (x \rightsquigarrow y \to x \nsim_i y)$ and $\forall x \forall y (x \sim_i y \to \neg x \rightsquigarrow y)$

These properties correspond to the interpretation of the relation $\rightsquigarrow$ as "is the next stage of the evolution of the world".

Definition 3 (semantics)**.** The truth of a modal formula φ in $\mathcal{M}^{\mathcal{E}\mathcal{E}}$ is defined as follows:
$\mathcal{M}^{\mathcal{E}\mathcal{E}}, w \vDash p \iff w \in V(p)$
$\mathcal{M}^{\mathcal{E}\mathcal{E}}, w \vDash \neg\varphi \iff \mathcal{M}^{\mathcal{E}\mathcal{E}}, w \nvDash \varphi$
$\mathcal{M}^{\mathcal{E}\mathcal{E}}, w \vDash \varphi \land \psi \iff \mathcal{M}^{\mathcal{E}\mathcal{E}}, w \vDash \varphi$ и $\mathcal{M}^{\mathcal{E}\mathcal{E}}, w \vDash \psi$
$\mathcal{M}^{\mathcal{E}\mathcal{E}}, w \vDash K_i\varphi \iff \forall w'(w \sim_i w' \to \mathcal{M}^{\mathcal{E}\mathcal{E}}, w' \vDash \varphi)$
$\mathcal{M}^{\mathcal{E}\mathcal{E}}, w \vDash {}^{\Box}X\varphi \iff \forall w'(w \rightsquigarrow w' \to \mathcal{M}^{\mathcal{E}\mathcal{E}}, w' \vDash \varphi)$
$\mathcal{M}^{\mathcal{E}\mathcal{E}}, w \vDash {}^{\Box}F\varphi \iff \forall \pi = (w \rightsquigarrow \ldots \rightsquigarrow w_n) \exists w'(w' \in \pi \land \mathcal{M}^{\mathcal{E}\mathcal{E}}, w' \vDash \varphi)$
$\mathcal{M}^{\mathcal{E}\mathcal{E}}, w \vDash {}^{\Box}G\varphi \iff \forall \pi = (w \rightsquigarrow \ldots \rightsquigarrow w_n) \forall w'(w' \in \pi \to \mathcal{M}^{\mathcal{E}\mathcal{E}}, w' \vDash \varphi)$

4. Scenario Generated EEM

Consider the procedure of how EEM models are generated by a Kripke model and an event model.

Definition 4 (event model)**.** $\mathcal{E} = (E, \{Q_i\}_{i\in A}, pre)$, where E is a set of events e, $Q_i \subseteq E \times E$ defined for every i in a set of agents A, $pre : E \mapsto \mathcal{L}_{EL}$, where $\mathcal{L}_{EL}$ is the basic propositional modal language with epistemic operator K_i (Moss 2015).

Definition 5 (product update)**.** $\mathcal{M} \otimes \mathcal{E} = (W', R', V')$, where $W' = \{(w, e) \mid \mathcal{M}, w \vDash pre(e)\}$, $(w, e)R'_i(w', e') \iff w \sim_i w' \land eQ_ie'$, $(w, e) \in V'(p) \iff w \in V(p)$ (Moss 2015).

Let us formulate the concept of **Scenario** (analogue of the protocol) that generates EEM models.

Definition 6 (Scenario)**.** $\mathbf{E}$ is a set of all event models $\mathcal{E}$. All possible event sequences are $\mathbf{E}^n$, where $n \in \mathbb{N}$. We need to identify such sequences of events that describe relevant changes in the world evolution. These relevant sequences are defined in the **Epistemic Scenario**. Let $\mathbf{S}$ denote such kind of scenario, $\mathbf{S} \subseteq \mathbf{E}^n$. The world's transformation by the event is denoted by the concatenation $we_n := (w, e_n)$.

The interaction of the Scenario and a Kripke model generates an evolutionary epistemic model (EEM).

Definition 7 (EEM generation)**.** $\mathcal{M} \otimes \mathbf{S} = (W', Root', \{\sim_i\}'_{i\in A}, \rightsquigarrow', V')$, where $\{\sim_i\}'_{i\in A}, V'$ are defined in the standard way, $W' = W \cup W \times E \cup W \times E \times E \cup \ldots \cup W \times E^n$, $Root' = W$ and $w \rightsquigarrow w' := w' \in W \times \mathcal{E}$ and $\exists \mathcal{E} : \exists e \in E \land w' = (w, e)$.

Consider a particular example for EEM.

5. An Example: The Envelope Show

There is a fictional Youtube show The Envelope, which involves two close friends. If private detectives manage to find out shocking facts about friends' relationships, they put a letter in an envelope with a detailed description of these facts. Otherwise, the envelope is empty. The show is based on the following scenario: first, one participant is chosen by lot who must leave the studio for a while, and the second participant has the right to see the contents of the envelope during this time. When the departing participant returns, no one tells him whether his close friend opened the envelope. Then the participants of the show draw lots again, which selects the one who will open the envelope in front of his friend but will not show him the contents of the envelope.

Figure 1 illustrates the EEM model for Ann and Boris participating in the show.

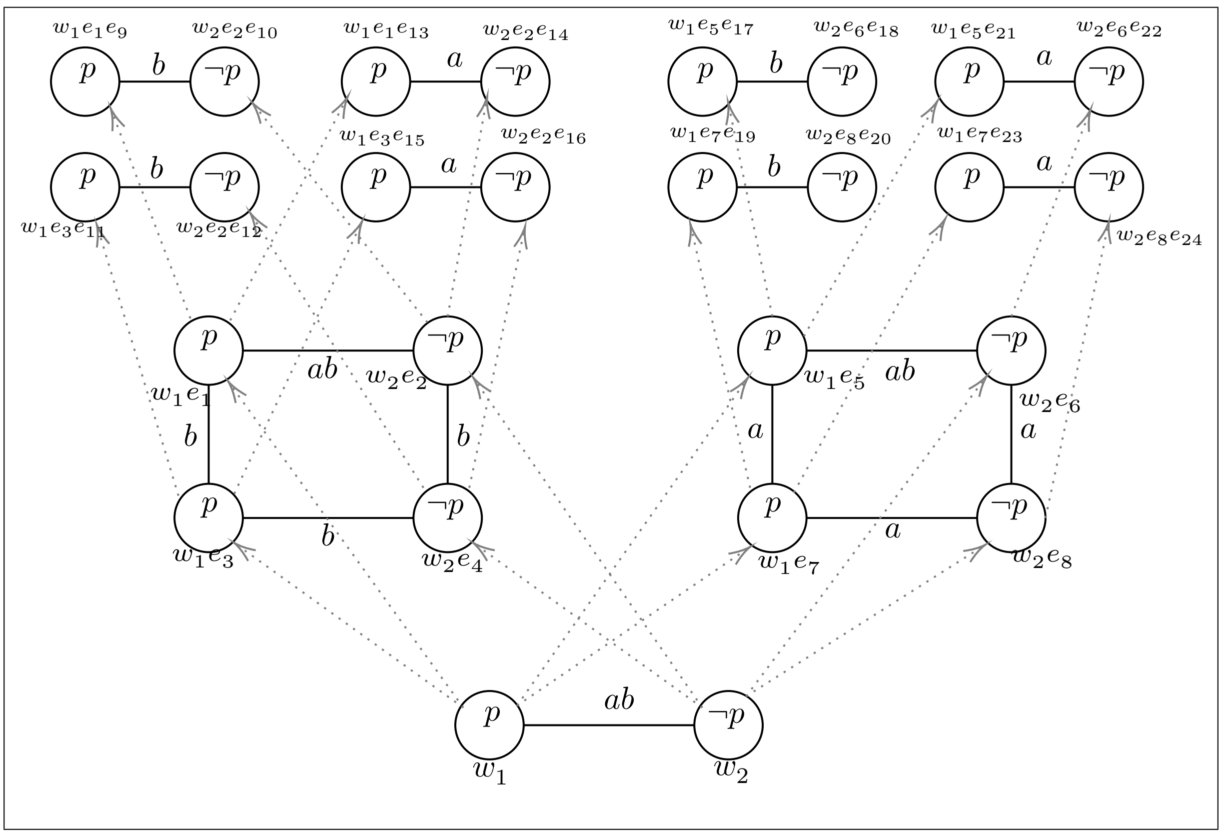

Figure 1: The Envelope Show

Let p ::= "there is a letter in the envelope" and $Root = \{w_1, w_2\}$.
The following formulas are true in Root worlds w_1, w_2.

$$\mathcal{M}, Root \vDash \hat{K}_b{}^{\Diamond}X^{\Diamond}XK_a p \tag{1}$$

$$\mathcal{M}, Root \vDash K_a K_b(\bigvee_{i\in\{a,b\}} {}^{\Box}FK_i^{?}p) \tag{2}$$

$$\mathcal{M}, Root \vDash \bigwedge_{i \in \{a,b\}} \neg K^?_i p \wedge (\Diamond X K^?_a p \wedge \Diamond X K^?_b p) \qquad (3)$$

Knowledge evolution in this example is described by an identical pattern. The agents do not know if the envelope is empty at the beginning of the show but at least one of them inevitably knows the content of the envelope at the second stage of the show.

6. Further Developments

The main goals for the future are to provide the complete axiomatization of EEM and investigate the problem of embedding EEM into related logics, such as temporal epistemic logic, dynamic epistemic temporal logic (Renne, Sack, Yap 2016), and others.

One of the key challenges in the development of our approach is the introduction of public announcement machinery in EEM structures.

The defined concept of the scenario may require further clarification. The introduced one does not include checking updates for compatibility, therefore the scenario can include a contradiction.

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