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LOGICAL ANALYSIS OF SYMBOLIC CONCEPTION REPRESENTATION IN TERMINOLOGICAL SYSTEMS

Abstract. Cognitive, or knowledge, agents, who are in some way aware of describing their own view of the world (based on their mental *concepts*), need to become concerned with the expressions of their own *conceptions*. My main supposition is that agents' conceptions are mainly expressed in the form of linguistic expressions that are spoken, written, and represented based on e.g. letters, numbers, or symbols. This research especially focuses on *symbolic conceptions* (that are agents' conceptions that are manifested in the form of their symbols). I attempt to logically (and using $\mathcal{CL}_{SYM}$) analyse symbolic conceptions in terminological systems. $\mathcal{CL}_{SYM}$ is an assertional fragment of my developed *Conception Language* ($\mathcal{CL}$), which is utilised as a formal-logical system for representing and explicating agents' conceptions of the world.

Keywords: assertional logic; cognitive agent; knowledge agent; conception language; conception; symbol; symbolic conception.

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1. Introduction

In the recent decades, *knowledge representation* (*KR*) in information and computer sciences has experienced significant improvements, see Brachman, Levesque 1985; Levesque, Brachman 1985; Harmelen, Lifschitz, Porter 2008. One of the main goals of KR systems and models can be interpreted to conceptualise and understand (about) conceptual relationships among various phenomena in the world and, subsequently, to logically describe those relationships. KR models have a special focus on *semantic knowledge*. Regarding semantic knowledge, some cognitive agent is concerned with concepts and

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the relations among them (see Turennout, Hagoort, Brown 1998). So, in fact, semantic knowledge can be seen to be constructed by concepts (as well as their (intra-/inter-) relationships).

For this reason, Description Logics (see Baader, Horrocks, (et al.) 2017; Sikos 2017) are adequate, as they are developed out of the attempt to represent semantic knowledge in terminological structures and to model an adequate formal semantics over terminological knowledge structures (see e.g. Badie 2017b; 2020b), in order to establish a common ground for cognition/knowledge for human beings, as well as for artificial agents. Any DL is a decidable fragment of predicate logic and DLs are a well-known family of knowledge-representation formalisms that are among the most widely used such formalisms in semantics-based systems. *Semantic networks* (which are knowledge bases that represent semantic interrelationships between various concepts; see Quillian 1968) and *frame-based systems* (based on which semantic knowledge becomes divided into interrelated sub-structural frames, in order to be represented; see Minsky 1974) are at the very foundations of DL. In addition, other logical representational systems based on *structural subsumption algorithms* (e.g., KRYPTON (see Brachman, Fikes, Levesque 1983), Kris (see Baader, Hollunder 1995)) have constructed supportive backgrounds for DL development. The frame language KL-ONE (see Brachman, Schmolze 1985) can be regarded as the very first DL-based logical system.

In this research, based on my research in Badie (2017b; 2020a,b), I focus on logical analysis of a fragment of my devised *Conception Language* ($\mathcal{CL}$), which is a kind of description logic. $\mathcal{CL}$ is utilised as a formal-logical system for representing and explicating cognitive, or knowledge, agents'[2] conceptions of the world. In other words, it is a *conception representation* formalism in terminological systems. The main contribution of this research is logical analysis of agents' conceptions which are manifested in the form of symbols. In more specific words, I will model the (assertional) conception language $\mathcal{CL}_{SYM}$ to represent agents' *symbolic conceptions* in terminological systems.

2. Concepts in Agents' Minds & Knowledge Bases

Over the years, the term *concept* has been used differently by many philosophers, psychologists, cognitive scientists, linguists, and computer scientists, see (Bartlett 1932; Allwood, Andersson 1976; Peacocke 1992; Hampton, Moss 2003; Margolis, Laurence 2007; 2010; Vicente, Martínez Manrique 2014; Margolis, Laurence 2015; Mahon 2015; Pinal 2016; Khalidi 2016; Cappelen 2018; Langland-Hassan, Vicente 2018; Gregory 2020).

In my view, concepts are mental and immaterial phenomena that are construed by agents in a particular state of consciousness and awareness, see (Badie 2017a; 2020a,b). Concepts are the consequences of agents' conceptualisation (of the world), as well as of

[2]Henceforth simply 'agents'.

their social linguistic interactions and communications, which all require some degree of awareness. In my approach, concepts are the primary/fundamental units of agents' terminological knowledge (see next section) and are the basic materials of what agents may/can/do mean by their descriptions of the world. Concepts are classifiable and serialisable in agents' minds as well as in their knowledge bases (see Badie 2017a). The most significant characteristic of concepts is that they are *identifiable* with the contents in agents', e.g., linguistic expressions, formal expressions, and/or numerical expressions.

3. Terminological Knowledge

Let $\mathcal{A}_{\mathcal{G}}$ stand for some agent. In my approach, $\mathcal{A}_{\mathcal{G}}$'s *terminological knowledge*, or formally '${}^{\mathcal{A}_{\mathcal{G}}}\mathcal{K}$', is an individual-based structural model of (semantic) knowledge. Any ${}^{\mathcal{A}_{\mathcal{G}}}\mathcal{K}$ is fundamentally modelled based on $\mathcal{A}_{\mathcal{G}}$'s conceptual-logical descriptions of how $\mathcal{A}_{\mathcal{G}}$ sees and interprets the interrelationships between $\mathcal{A}_{\mathcal{G}}$'s two, or more, conceptions[3] of the world. Actually, by building their terminological knowledge, agents attempt to satisfy their own semantical (knowledge-based) rules based on their conceptualisations of logical relationships between their various descriptions of the world.

4. Agents' Conceptions of the World

In this research, I am going to logically deal with agents' descriptions of the world (which are believed to be expressed based on their mental concepts) in terminological systems. For this purpose, I draw on *nominal conceptualism* (see my research in Badie 2020a,b). Summing up 'nominal conceptualism', agents, who are in some way aware of expressing their own descriptions of the world, become concerned with the production of their *conceptions*. Therefore, I regard agents' conceptions as linguistic outcomes of their constructed concepts. It is an underlying research hypothesis of this paper that the most fundamental building blocks of agents' descriptions of the world are expressible based on their conceptions.

4.1. Agents' Symbolic Conceptions

Agents who are going to express their own conceptions, deal with their linguistic expressions that are spoken, written, and represented based on, e.g., letters, numbers, *symbols*[4]. Actually, it can be interpreted that 'symbols' are those (kinds of) 'representations' whose relation to their original concepts (in the mind or knowledge base) is an imputed (and recognised) character. In this research, *symbolic conceptions* are

[3] The next section will especially focus on the notion of 'conception'.

[4] See Dewey 1946; Atkin 2010 for more detailed information on whatness of symbols in *semiotic theory* of Charles Sanders Peirce.

conceptions which are manifested and presented in the form of symbols. In other words, symbolic conceptions are conceptions that are are *symbolically* expressed and represented. It shall be taken into account that all symbolic conceptions are surely conceptions, but all conceptions are not necessarily symbolic.

5. Representing Symbolic Conceptions Using $\mathcal{CL}_{SYM}$

In this section I attempt to model the conception language $\mathcal{CL}_{SYM}$ in order to analyse assertional representation of symbolic conceptions in terminological systems. $\mathcal{CL}_{SYM}$ is structured in terms of the non-logical constructors *conception* (formally: '$\mathcal{C}$') and *symbol* (formally: '$\mathcal{S}$'). Conceptions are either *monadic* or *polyadic*. Monadic and polyadic conceptions are assertionally represented by ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}(\mathcal{S})$ and ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}(\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_n)$, respectively (where $n \geq 2$).

The vocabulary of $\mathcal{CL}_{SYM}$ contains the logical symbols *braces* (i.e. '{', '}') and *parentheses* (i.e. '(', ')'). Braces represent the concept of *non-ordered collection* (of symbols). Also, parentheses represent *ordered collection* (of symbols).

In order to semantically analyse $\mathcal{CL}_{SYM}$, I need to utilise agents' *semantic interpretations*.

Definition 1. Some agent's *semantic interpretation* (formally: '${}^{\mathcal{A}_\mathcal{G}}\mathcal{J}$') is defined based on the following ingredients:

i. A non-empty set (formally: '${}^{\mathcal{A}_\mathcal{G}}\Delta$') that is agent's *interpretation domain* and consists of all symbols that exist in (and thus can be expressed by means of) $\mathcal{A}_\mathcal{G}$'s symbolic conceptions.

ii. Agent's *interpretation function* (formally: '${}^{\mathcal{A}_\mathcal{G}}\mathcal{J}$') that assigns (1) every individual symbol to its interpreted one, and (2) every collection of n symbols to a collection of n interpreted symbols. More specifically, regarding ${}^{\mathcal{A}_\mathcal{G}}\mathcal{S}$ (as well as all ${}^{\mathcal{A}_\mathcal{G}}\mathcal{S}_i$; where $i \in [1, n)$) as symbols in $\mathcal{A}_\mathcal{G}$'s mind or knowledge base, we will have:

   1. ${}^{\mathcal{A}_\mathcal{G}}\mathcal{S}^\mathcal{J} \in {}^{\mathcal{A}_\mathcal{G}}\Delta^\mathcal{J}$, and
   2. $({}^{\mathcal{A}_\mathcal{G}}\mathcal{S}_1^\mathcal{J}, {}^{\mathcal{A}_\mathcal{G}}\mathcal{S}_2^\mathcal{J}, \dots, {}^{\mathcal{A}_\mathcal{G}}\mathcal{S}_n^\mathcal{J}) \subseteq {}^{\mathcal{A}_\mathcal{G}}\Delta^\mathcal{J} \times {}^{\mathcal{A}_\mathcal{G}}\Delta^\mathcal{J} \times \dots \times {}^{\mathcal{A}_\mathcal{G}}\Delta^\mathcal{J}$.

Proposition 1. The monadic symbolic conception ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}(\mathcal{S})$ expresses that some agent $\mathcal{A}_\mathcal{G}$ has conceptualised that their conception $\mathcal{C}$ is (or can be) manifested in the form of the symbol $\mathcal{S}$. Semantically we have: $\mathcal{S}^\mathcal{J} \in {}^{\mathcal{A}_\mathcal{G}}\mathcal{C}^\mathcal{J}$.

According to $\mathcal{S}^\mathcal{J} \in {}^{\mathcal{A}_\mathcal{G}}\mathcal{C}^\mathcal{J}$, it is interpreted (by $\mathcal{A}_\mathcal{G}$) that $\mathcal{S}$ (which has been conceptualised and interpreted by $\mathcal{A}_\mathcal{G}$) belongs to their interpreted $\mathcal{C}$.

For example, in a terminological system, ${}^{John}Leaf(\clubsuit)$ can express that John conceptualises and interprets that the symbol '♣' represents his conception of the concept LEAF. Semantically we have: ${}^{John}\clubsuit^\mathcal{J} \in {}^{John}Leaf^\mathcal{J}$.

Proposition 2. The polyadic symbolic conception ${}^{\mathcal{A}_{\mathcal{G}}}\mathcal{C}(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_n)$ expresses that some agent $\mathcal{A}_{\mathcal{G}}$ has conceptualised that all the symbols $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_n$ are related (or relatable) to each other by means of their conception ${}^{\mathcal{A}_{\mathcal{G}}}\mathcal{C}$. Semantically we have: $(\mathcal{S}_1^{\mathcal{I}}, \mathcal{S}_2^{\mathcal{I}}, \ldots, \mathcal{S}_n^{\mathcal{I}}) \subseteq {}^{\mathcal{A}_{\mathcal{G}}}\mathcal{C}^{\mathcal{I}}$.

Taking into consideration $(\mathcal{S}_1^{\mathcal{I}}, \mathcal{S}_2^{\mathcal{I}}, \ldots, \mathcal{S}_n^{\mathcal{I}}) \subseteq {}^{\mathcal{A}_{\mathcal{G}}}\mathcal{C}^{\mathcal{I}}$, it is conceptualised and interpreted (by $\mathcal{A}_{\mathcal{G}}$) that the collection of their interpretations of n symbols becomes included in their interpretation of $\mathcal{C}$.

For example, Mary conceptualises that all the symbols '○', '⊙', '•', '⊛', and '⋇' represent what she conceptualises as CIRCLE. In other words, '○', '⊙', '•', '⊛', and '⋇' are correlatable to each other (by Mary) and by means of her conception of 'being circle'. Consequently we will have ${}^{Mary}Circle(\bigcirc, \odot, \bullet, \circledast, \divideontimes)$ in our terminological system. Then, semantically we have: $({}^{Mary}\bigcirc^{\mathcal{I}}, {}^{Mary}\odot^{\mathcal{I}}, {}^{Mary}\bullet^{\mathcal{I}}, {}^{Mary}\circledast^{\mathcal{I}}, {}^{Mary}\divideontimes^{\mathcal{I}}) \subseteq {}^{Mary}Circle^{\mathcal{I}}$. In addition, it can be interpreted that any of these symbols belongs to her conception of CIRCLE. Therefore, we would formally have five monadic symbolic conceptions as follows: (1) ${}^{Mary}Circle(\bigcirc)$, (2) ${}^{Mary}Circle(\odot)$, (3) ${}^{Mary}Circle(\bullet)$, (4) ${}^{Mary}Circle(\circledast)$, and (5) ${}^{Mary}Circle(\divideontimes)$. Semantically: ${}^{Mary}\bigcirc^{\mathcal{I}} \in {}^{Mary}Circle^{\mathcal{I}}$, ${}^{Mary}\odot^{\mathcal{I}} \in {}^{Mary}Circle^{\mathcal{I}}$, ${}^{Mary}\bullet^{\mathcal{I}} \in {}^{Mary}Circle^{\mathcal{I}}$, ${}^{Mary}\circledast^{\mathcal{I}} \in {}^{Mary}Circle^{\mathcal{I}}$, and ${}^{Mary}\divideontimes^{\mathcal{I}} \in {}^{Mary}Circle^{\mathcal{I}}$.

Definition 2. The logical operator '${}^{\mathcal{A}_{\mathcal{G}}}\top$' is called *agent's top conception.* It stands for some agent's conception of the logical concept of *tautology.* In other words, ${}^{\mathcal{A}_{\mathcal{G}}}\top$ expresses $\mathcal{A}_{\mathcal{G}}$'s conception of TRUTH (based on their own conceptualisation of the world). Semantically expressing, by dealing with $\top$, $\mathcal{A}_{\mathcal{G}}$ interprets all elements of their interpretation domain.

Definition 3. The logical operator '${}^{\mathcal{A}_{\mathcal{G}}}\bot$' is called *agent's bottom conception.* It stands for some agent's conception of the logical concept of *contradiction.* Actually, ${}^{\mathcal{A}_{\mathcal{G}}}\bot$ expresses $\mathcal{A}_{\mathcal{G}}$'s conception of FALSITY (based on their own conceptualisation of the world). Semantically expressing, by dealing with $\bot$, $\mathcal{A}_{\mathcal{G}}$ interprets something that is not located in their interpretation domain. In fact, ${}^{\mathcal{A}_{\mathcal{G}}}\bot$ interprets nothing.

Relied on the offered definitions and propositions, table 1 presents the syntax and semantics of the language $\mathcal{CL}_{SYM}$.

Syntax	**Semantics**
${}^{\mathcal{A}_{\mathcal{G}}}\top$	${}^{\mathcal{A}_{\mathcal{G}}}\Delta^{\mathcal{I}}$
${}^{\mathcal{A}_{\mathcal{G}}}\bot$	$\emptyset$
${}^{\mathcal{A}_{\mathcal{G}}}\mathcal{C}(\mathcal{S})$	$S^{\mathcal{I}} \in {}^{\mathcal{A}_{\mathcal{G}}}\mathcal{C}^{\mathcal{I}}$
${}^{\mathcal{A}_{\mathcal{G}}}\mathcal{C}(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_n)$	$(\mathcal{S}_1^{\mathcal{I}}, \mathcal{S}_2^{\mathcal{I}}, \ldots, \mathcal{S}_n^{\mathcal{I}}) \subseteq {}^{\mathcal{A}_{\mathcal{G}}}\Delta^{\mathcal{I}} \times {}^{\mathcal{A}_{\mathcal{G}}}\Delta^{\mathcal{I}} \times \cdots \times {}^{\mathcal{A}_{\mathcal{G}}}\Delta^{\mathcal{I}}$

Table 1: $\mathcal{CL}_{SYM}$ Syntax and Semantics

6. $\mathcal{CL}_{SYM}$-based Analysis of Symbolic Conceptions in Terminological Systems

In order to analyse a logical-terminological system based on some agent's symbolic conceptions of the world, the following factors should be taken into consideration:

1. Let $\mathcal{S}$ (for example: '≀') be some arbitrary symbol in the world and be independent from any specific ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}$. Also, let it be absolutely distinct (i.e. it has no connection to any other symbol in the world). Accordingly, such an existence is an individual symbol (with the relation of valence 0) in our logical-terminological system.

2. Suppose that $\mathcal{A}_\mathcal{G}$ conceptualises that the symbol '△' is (or can be) an instance of their conception ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}_1$. Thereby $\mathcal{A}_\mathcal{G}$ would have the symbolic conception $\mathcal{C}_1(\triangle)$. In fact, $\mathcal{A}_\mathcal{G}$'s conceptualisation has supported the logical construction of the relationship '${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}_1 \longleftrightarrow {}^{\mathcal{A}_\mathcal{G}}\triangle$' that is a relation of valence 1. Such a conceptualisation has supported $\mathcal{A}_\mathcal{G}$ through a process of classifying and, subsequently, of relating their conceptualised '△' to their conception $\mathcal{C}_1$.

3. Accept that $\mathcal{A}_\mathcal{G}$ conceptualises that the symbols '•' and '◦' are (or can be) related to each other by means of their conception ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}_2$. In fact, $\mathcal{A}_\mathcal{G}$ would have the symbolic conception ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}_2(\bullet, \circ)$. In more specific words, $\mathcal{A}_\mathcal{G}$'s conceptualisation has supported the logical construction of the relation '${}^{\mathcal{A}_\mathcal{G}}\bullet \longleftrightarrow {}^{\mathcal{A}_\mathcal{G}}\mathcal{C}_2 \longleftrightarrow {}^{\mathcal{A}_\mathcal{G}}\circ$' that is a relation of valence 2. Such a conceptualisation has supported $\mathcal{A}_\mathcal{G}$ through a process of classifying and, subsequently, of relating their conceptualised '•' and '◦' together by means of their conception $\mathcal{C}_2$.

My most central assumption is that some agent's symbolic conception (as well as their any other conception) is conceptually and logically correlated with a (distinct) conceptual entity (in their mind or knowledge base). Any symbolic conception can also be regarded as a class of conceptual entities. A logical model of symbolic conception representation must be able to represent terminological knowledge in terms of (1) symbolic conceptions and (2) (symbolic) conceptions' interrelationships, see Badie 2020a. In fact, symbolic conceptions and their interrelationships create terminologies in order to represent terminological knowledge.

Figure 1 presents the logical interrelationships among key concepts in this research. This semantic network can be interpreted a schematic ontology[5] for the analysis of symbolic conceptions.

[5]In this research an ontology is a conceptual and logical specification of shared conceptualisation in a specific domain of discourse.

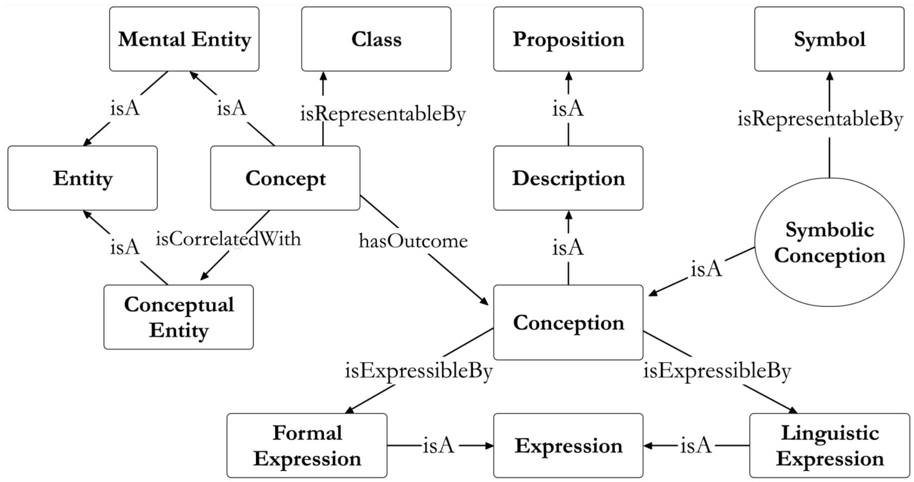

Figure 1: Ontology for symbolic conception analysis

6.1. $\mathcal{CL}_{SYM}$-based Analysis of an Example

Let Ann be a student and express her conceptions of the concepts MULTIPLICATION and DIVISION by the symbols '$\boxtimes$' and '$\div$', respectively. We will have the following consequences:

- Ann's monadic symbolic conceptions can be represented by $\mathcal{CL}_{SYM}$-based descriptions $^{Ann}Multiplication(\boxtimes)$ and $^{Ann}Division(\div)$.
- Any of Ann's symbolic conceptions $^{Ann}Multiplication(\boxtimes)$ and $^{Ann}Division(\div)$ is conceptually correlated with a [distinct] mental entity (in her mind).
- The symbols '$\div$' and '$\boxtimes$' are conceptualised by Ann to be the *symbolic instances* of her conceptions of the concepts DIVISION and MULTIPLICATION, respectively.
- Ann's symbolic conception $^{Ann}Division(\div)$ has a strong correlation with her some linguistic expressions (e.g., 'division', 'separation', 'split').
- Ann's symbolic conception of the concept DIVISION can also be represented in the form of a class/collection of correlated symbols, e.g., $\{^{Ann}\div,\ ^{Ann}/,\ ^{Ann}|\}$; where the logical symbols '{' and '}' are utilised in order to represent a class/collection of the (non-logical) individual symbols '$^{Ann}\div$', '$^{Ann}/$', and '$^{Ann}|$' (that are in Ann's mind).
- According to Ann's symbolic conception of MULTIPLICATION, the symbol '$\boxtimes$' is transformable into $Multiplication(\boxtimes)$, and vice versa.
- According to Ann's symbolic conception of DIVISION, the junctions between the symbols '$\div$' and '$|$' are transformable into the concept DIVISION (and vice vera).
- According to Ann's symbolic conception of DIVISION, there would be an *identifying* relationship between '$^{Ann}\div$' and '$^{Ann}|$' (as well as between the symbolic conceptions $^{Ann}Division(\div)$ and $^{Ann}Division(|)$).

- According to Ann's symbolic conceptions of MULTIPLICATION and DIVISION, the interrelationships between '${}^{Ann}\div$' and '${}^{Ann}\boxtimes$' are transformable into some conceptual junctions between her symbolic conceptions ${}^{Ann}\mathit{Division}(\div)$ and ${}^{Ann}\mathit{Multiplication}(\boxtimes)$, and vice versa.
- According to Ann's symbolic conception of DIVISION, '${}^{Ann}\div$' has a property/attribute (e.g., 'being divider', 'being separator'). It can be interpreted that the property 'being divider' is describable based on the conceptual (intra-)relationships between '${}^{Ann}\div$' and itself (as well as between the symbolic conception ${}^{Ann}\mathit{Division}(\div)$ and itself).

Let ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}_1(\mathcal{S}_1)$ and ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}_2(\mathcal{S}_2)$ be two monadic symbolic conceptions. It shall be emphasised that there is no way to model a terminological knowledge based on the collection(s) of ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}_1(\mathcal{S}_1)$ and ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}_2(\mathcal{S}_2)$ when they are—conceptually and logically—irrelevant to, and independent of, each other. For example, we cannot model a terminological knowledge based on the monadic symbolic conceptions ${}^{Bob}\mathit{Dagger}(\dagger)$ and ${}^{Bob}\mathit{Square}(\square)$ when ${}^{Bob}\dagger$ and ${}^{Bob}\square$ have—conceptually and logically (and based on Bob's conceptualisation)—been independent.

Proposition 3. Conceptualising and interpreting, by $\mathcal{A}_\mathcal{G}$, that there is/are conceptual and logical interconnection(s) between their two (or more) monadic symbolic conceptions of the world, can certainly constitute a building block of a terminological knowledge (about those symbolic conceptions) in $\mathcal{A}_\mathcal{G}$'s mind or knowledge base.

Now accept that there is/are conceptual and logical interconnection(s) between the monadic symbolic conceptions ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}_3(\mathcal{S}_3)$ and ${}^{\mathcal{A}_\mathcal{G}}\mathcal{C}_4(\mathcal{S}_4)$ in $\mathcal{A}_\mathcal{G}$'s mind or knowledge base. For example, ${}^{Sara}\mathit{Triangle}(\triangle)$ and ${}^{Sara}\mathit{Circle}(\bigcirc)$ are two monadic symbolic conceptions of Sara. Regarding Sara's conceptualisation of the world, these two symbolic conceptions are—conceptually and logically—relevant to each other and can collectively construct a (semantic and terminological) knowledge of *geometrical shapes* in her mind. Actually, ${}^{Sara}\mathit{Triangle}(\triangle)$ and ${}^{Sara}\mathit{Circle}(\bigcirc)$ are terminologically correlated with the expressions 'geometrical shape', 'geometrical', 'shape' (as well as with some other possible linguistic expressions). Similarly, based on Sara's conceptualisation of the world, we can represent the polyadic symbolic conceptions ${}^{Sara}\mathit{GeometricalShapes}(\triangle, \bigcirc)$ and ${}^{Sara}\mathit{GeometricalShapes}(\triangle, \bigcirc, \lozenge, \square)$. Considering these polyadic symbolic conceptions, Sara's conception of the concept GEOMETRICAL-SHAPES can be represented in the form of the collection $\{{}^{Sara}\triangle, {}^{Sara}\bigcirc, {}^{Sara}\lozenge, {}^{Sara}\square\}$. In this example, formally we have:

$$
\begin{aligned}
{}^{Sara}\triangle^{\mathcal{I}} &\in {}^{Sara}\mathit{Triangle}^{\mathcal{I}},\\
{}^{Sara}\bigcirc^{\mathcal{I}} &\in {}^{Sara}\mathit{Circle}^{\mathcal{I}},\\
({}^{Sara}\triangle^{\mathcal{I}}, {}^{Sara}\bigcirc^{\mathcal{I}}) &\subseteq {}^{Sara}\mathit{GeometricalShapes}^{\mathcal{I}},\\
({}^{Sara}\triangle^{\mathcal{I}}, {}^{Sara}\bigcirc^{\mathcal{I}}, {}^{Sara}\lozenge^{\mathcal{I}}, {}^{Sara}\square^{\mathcal{I}}) &\subseteq {}^{Sara}\mathit{GeometricalShapes}^{\mathcal{I}}.
\end{aligned}
$$

Subsequently we have:[6]

$$
\begin{aligned}
&{}^{Sara}\mathcal{J} \models {}^{Sara}Triangle(\triangle),\\
&{}^{Sara}\mathcal{J} \models {}^{Sara}Circle(\bigcirc),\\
&{}^{Sara}\mathcal{J} \models {}^{Sara}GeometricalShapes(\triangle, \bigcirc),\\
&{}^{Sara}\mathcal{J} \models {}^{Sara}GeometricalShapes(\triangle, \bigcirc, \Diamond, \Box).
\end{aligned}
$$

7. Logical Modelling of Terminological Knowledge Based on Agents' Symbolic Conceptions

Thereby it shall be concluded that:

$$
\begin{aligned}
&{}^{Sara}\mathcal{J} \models {}^{Sara}\mathcal{K}\\
&\qquad \&\\
&{}^{Sara}\mathcal{K} \models \{{}^{Sara}Triangle(\triangle), {}^{Sara}Circle(\bigcirc),\\
&\qquad {}^{Sara}GeometricalShapes(\triangle, \bigcirc), {}^{Sara}GeometricalShapes(\triangle, \bigcirc, \Diamond, \Box)\}.
\end{aligned}
$$

The final conclusion expresses the fact that ${}^{Sara}\mathcal{J}$ is a (*semantic*) *model* for Sara's own expressed monadic and polyadic symbolic conceptions. Also, ${}^{Sara}\mathcal{J}$ semantically satisfies Sara's terminological knowledge (or $\mathcal{K}$), where $\mathcal{K}$ semantically satisfies the collection of her monadic and polyadic symbolic conceptions.

Taking into consideration Sara's terminological knowledge, any of '${}^{Sara}\triangle$', '${}^{Sara}\bigcirc$', '${}^{Sara}\Diamond$', and '${}^{Sara}\Box$' (as well as any of their conceptual/logical interrelationships) is transformable into her mental concept GEOMETRICAL-SHAPES, and vice versa. Correspondingly, regarding Sara's symbolic conception of GEOMETRICAL-SHAPES, any of '${}^{Sara}\triangle$', '${}^{Sara}\bigcirc$', '${}^{Sara}\Diamond$', and '${}^{Sara}\Box$' becomes conceptualised (by Sara) to have the property *BeingGeometricalShape* and, inductively, the properties *BeingShape* and *BeingGeometrical*. It shall be interpreted that both *BeingShape* and *BeingGeometrical* are formed based on the (intra-)relationships between any of '${}^{Sara}\triangle$', '${}^{Sara}\bigcirc$', '${}^{Sara}\Diamond$', '${}^{Sara}\Box$' and themselves (under Sara's conception of the concept GEOMETRICAL-SHAPES).

Conclusions and Future Research

This research has focused on the logical analysis of cognitive, or knowledge, agents' symbolic conceptions of the world in terminological systems. In more specific words, the research has attempted to model and utilise the assertional conception language $\mathcal{CL}_{SYM}$ in order to, logically and terminologically, deal with symbolic conceptions.

[6]The formal expression '... $\models {}^{\mathcal{A}_\mathcal{G}}\mathcal{C}(\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_n)$' reads '$\mathcal{A}_\mathcal{G}$'s symbolic conception $\mathcal{C}(\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_n)$ holds (i.e., is valid and meaningful) in $\mathcal{CL}_{SYM}$'.

$\mathcal{CL}_{SYM}$ is a fragment of the Conception Language ($\mathcal{CL}$) which I have developed as a formal-logical system for representing and explicating agents' conceptions of the world.

In my forthcoming research Badie (forthcoming), I will model a *Qualified Conception Language* (*QCL*) as a terminological conception representation model (in the world of *qualified* and *contingent* information). The research Badie (forthcoming) will conceptualise agents' conceptions based on qualified information in order to logically express how the concepts of *possibility* and *necessity* can be analysed in $\mathcal{CL}$-based descriptions of agents' conceptions. Accordingly, I will model the formal language $\mathcal{QCL}$ (the conception language with contingent symbols).

The outcomes of the current research and of Badie (forthcoming) can collectively support my future research in qualified analysis of agents' symbolic conceptions in terminological systems.

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