Is the Necessary Someway Possible?

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Abstract. The aim of this essay is to reexamine an argument by Aristotle (De Interpretatione) and its reconstruction by Jan Lukasiewicz, who used the tools of modern logic. The Modal Logic has been the object of refined analysis by Jan Lukasiewicz. He fulfilled this project in his classic book, Aristotle's Syllogistic. He introduced modal "functors" to reconstruct Aristotle's modal logic. I will discuss Lukasiewicz's way to solve the questions concerning Aristotle's modal logic.

Consequently, I will present a new symbolic representation of propositions concerning the modalities of necessity and possibility, and I will try to discuss the ontological status of such propositions and to answer the question: is the necessary someway possible? DOI: 10.52119/LPHS.2024.48.54.012.

Russian logician A. M. Anísov once asked: how is the possible possible? Learning from or inspired by him, maybe we can ask, if the necessary is someway possible? In fact, we could write that question another way. Maybe it would be strange for someone how I will formulate the same question:

is there possibility beyond the contingency?

I will try demonstrating it thinking about Aristotle's modal logic and Jan Lukasiewics's work about that subject.

Aristotle's modal logic was translated by Jan Lukasiewicz in Chapter VI of "Aristotle's Syllogistic," employing the symbolism of modern logic. As he himself tells us:

Two of the modal terms, 'necessary' and 'possible,' and their interrelations, are of fundamental importance. In *De Interpretatione*, Aristotle mistakenly asserts that possibility implies non-necessity, i.e., in our terminology:

(1) If it is possible that p, it is not necessary that p. He later sees that this cannot be right, because he accepts that necessity implies possibility, i.e.:

(2) If is necessary that p, it is possible that p, and from (1) and (2) there would follow, by the hypothetical syllogism, that

(3) If it is necessary that p, it is not necessary that p, which is absurd.

We must note that conclusion (3), "If it is necessary that p, it is not necessary that p" is labelled by Aristotle and by Lukasiewicz as "absurd." Aristotle himself used the word "atopos." We can translate "atopos" as something that cannot take place; that cannot be. This is limited to explaining a result which cannot be admitted. Aristotle at this moment doesn't support any conclusion at all, in contrast with the explosion of contradictions in *Metaphysics*,[14] as we shall see below. It is interesting to note as well that Aristotle not only discusses and discovers the modalities as logical functions; he also attempts to situate them in connections, through mechanisms of conversion, inversion, deduction, implication and reduction.

The great merit of Aristotle in modal logic is the discovery of relations between different concepts of modalities. How may these different modalities be connected with each other, despite the fact that they designate well-marked fields which appear as irreducible modalities?

The clearest definition of the modality "possible" is the following: "Something that can be or not be, like the clothes which can be cut or not be cut" (19a 10–15).

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[14]1007b 20–25.

I consider this presentation of the possible to be very important to any logical project about possibility.

Despite the force of Lukasiewicz's demonstration, which we saw above, and of his rewriting of Aristotle's text, the "absurd" derived by both of them, one in antiquity and the other in our own times, doesn't fit in with the general sense of De Interpretatione, which was established beyond doubt in the experience of the fatalist's argument in the ninth chapter of the book. There is in this passage a real error. Aristotle probably committed that mistake, an inconsistency with his own work, for the sake of conciseness of writing. He wrote the modal statements in abbreviated formulae.

He did not write the modal propositions textually, nor did he write a statement-like nuclei less incompletely. For example (22a 24–27), he wrote:

"possible to be";

"admissible to be";

"not impossible to be";

"not necessary to be."

Unfortunately, some modal statements have proper formulae and nuances, which—if they are forgotten—lead us almost necessarily, if not necessarily, to error. We will see later the nature of these errors.

Jan Lukasiewicz's exposition of Aristotle's modal propositions follows Chapters XII and XIII of *De Interpretatione*. Nevertheless, the ninth chapter is very important for resolving the question, particularly with regard to the formal representation of modal statements in Aristotle's logical thought.

In the ninth chapter (IX), the fatalist, in order to demonstrate the unsound nature of the possibility of a third logical value, presents the following argument:[15] if for two contradictory statements (a) and (b), the two values—True (T) and False (F)—are not sufficient:

(a)   Tomorrow there will be a sea battle near Salamina Island;

and

(b)   Tomorrow there won't be a sea battle near Salamina Island.

Then we will have for these two propositions a third value, which will be simultaneously the negation of both proposition (a) and proposition (b); this new value will be neither true ($T/1$) nor false ($F/0$). In this way, the fatalist comes to the following conclusion:

(4) "It is necessary that tomorrow neither will there be a sea battle near Salamina Island nor will there **not** be a sea battle near Salamina Island. Something like this: $L\sim(p \text{ and } \sim p)$, where p, as a value, will be 1 or 0, $\sim p$, 1 or 0; and $[L\sim(p \text{ and } \sim p)]$ will be something like $1/2$ (neither one nor zero — $1/2$).[16]

Despite the fact that this argument was taken as absurd (there can be no $1/2$ because it makes no sense), the fatalist's experiment reveals practically what we know as modal logic.

Aristotle was to basically only correct the fatalist concerning the use of the "functor" or "connective" (*and*) to express the necessity. The error of the fatalist consists in the articulation of the "functor" *and* (*neither... nor = not... and not...*) with the modality "necessary". The possibility (other basic modality) will be articulated with the same two statements (p and $\sim p$), but with the functor *and*". Here, from the exposition of the ninth chapter, we come to the following equivalence as fundamental in Aristotle's modal logic:

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[15]The exposition is merely schematic here.

[16]Ackrill translated (18b 24–25): "Take a seabattle: it would have neither to happen nor not happen."

(5) $L(p \text{ or } \sim p) \equiv M(p \text{ and } \sim p)$.

For the future, the contradictions are secured, as we know by the semantical exposition in *De Interpretatione*.[17] The contradiction of two propositions in the future is one exception to the rule of contradictory pair (RCP):[18]

> Clearly, then, it is not necessary that of every affirmation and opposite negation one should be true and the other false. For what holds for things that are does not hold for things that are not but may possibly be or not be; with these it is as we have said.

As we know, by the RCP, one proposition of the contradictory pair (if the pair has singular propositions) should be true and the other false; and this law concerns the hypothetical necessity. In fact, hypothetical necessity here means past and present.

We must assume that contradictions for sentences about the future don't run the risk of triviality or inconsistency. Why? Because contradictions concerning the future appear in modal logic like the contradictions would appear afterwards in Vasil'ev's [2] indifferent judgment type of thought. They are intrastatemental contradictions and not interstatemental contradictions. They refer to the *dictum* and not to the modalities where the *dictum* is. This means that Aristotle discovered this crucial distinction: *it is one thing to think the intrastatemental contradictions, it is another thing to think the interstatemental contradictions*.

For example:

Indifferent judgment in Vasil'ev: *A is B and $\sim B$*.

Modal sentence in Aristotle: *p and $\sim p$ is possible*.

In both of the above cases the kind of contradiction doesn't put the system at risk.

The modal system, meanwhile, doesn't admit the following kind of contradiction:

$\sim M(p.\sim p).M(p.\sim p)$, which is in essence $\sim M.M$.

In fact, the modal proposition is $M(p.\sim p)$ and not Mp or $M\sim p$, because p and $\sim p$ have the same ontological status and consequently cannot be separated.

At the limit, the cause of the individuation of sentences like *S is P* and *S is not P* is the different ontological status that one has vis-à-vis the other. It doesn't happen with $M(p.\sim p)$, because p and $\sim p$ form a unity, as they have the same ontological status. One proposition expresses a fact of just one ontological status, and no more.[19]

The abbreviated formulae of modal logic presented in Chapters XII and XIII of *De Interpretatione* must be read as the ninth chapter tells us. Consequently, I propose to rebuild Aristotle's argument and Jan Lukasiewicz's formulations about this argument in the following way:

(6) (If it is possible that p and that non-p), it is not necessary that p and that non-p;

(7) If it is necessary that p or that non-p, (it is possible that p and non-p).

From (6) and (7), we must cut the middles represented by the brackets (()), in the hypothetical syllogism; then we will obtain the following result:

(8) If it is necessary that p or that non-p, it is not necessary that p and that non-p;

This implication could be expressed in the following way:

(9) If it is necessary that p or that non-p, it is possible that p and that non-p.

Then (9) is (5), and we have obtained a sound result:

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[17] Ackrill's translation of *De Interpretatione* in 19a 39 — 19b 4.

[18] RCP: about this rule, see [1], Chapter 9, "The third exception to the RCP."

[19] We must agree that this is not the place to discuss an interesting question, what is really a proposition?

$L(p$ or $\sim p)$ is equivalent to $M(p$ and $\sim p)$.

This rebuilding of the argument shows us that the absurd obtained by Aristotle and Jan Lukasiewicz doesn't subsist if we make a correct representation of the modal proposition or statement. The absurd doesn't subsist because the modal functions (necessary and non-necessary), which appear on both sides of the statement obtained by (1) and (2), are linked to different connectors or functors, on the one side "or" (disjunction) and on the other "and" (conjunction).

The argument is sound if it conserves the necessary articulation between functors and modalities.

On the other hand, assuming the necessary articulation between functors and modalities, we must identify what is the overall time frame governing the modality. This is another very important discovery of Aristotle's: *tempus regit modum*. Therefore, concerning the past, we can say:

It is necessary that p or that non-p,[20] but in this case the sentence "it is necessary that p or that non-p" in fact hides two sentences: 1) "it is necessary that p" and 2) "it is necessary that non-p." The reason is their different ontological status: one of the two sentences should be true, and the other, false. Here, as the existence of the past guarantees the distributivity of the necessity, we may write the following formulae:

$$L(p \text{ or non-}p) \equiv Lp \text{ or } L\sim p // \sim M(p \text{ or non-}p) \equiv \sim Mp \text{ or } \sim M\sim p$$

$M(p$ and $\sim p)$ is not sound here, because the possible, as contingency, is a modality exclusive to the future, but we, for the past, can admit $\sim M$ and its consequences; therefore,

$Lp \equiv \sim M\sim p$;

$L\sim p \equiv \sim Mp$;

$\sim\sim M\sim p \equiv \sim Mp$, where $\sim\sim M$ is not M but L;

$\sim\sim Mp \equiv \sim M\sim p$, where $\sim\sim M$ is not M but L.

It is evident that most likely Chapters XII and XIII of *De Interpretatione* were written a long time after Chapter IX. This may explain the fact that in rewriting the book, Aristotle wrote schematic formulae which could not express the results obtained in Chapter IX. Lukasiewicz repeated this error. This is an error of symbolic representation. In fact, if it concerns the future, possibility should be represented by the following sentence: "It is possible that p and that non-p," because p and non-p have the same ontological status. We cannot separate them because *dictum* and *modality* form one single sentence. Concerning the future, we write: "It is necessary that p or that non-p." An interrelation between possibility and necessity should exhibit the following equivalence as one of its axioms, if we want to be faithful to the ninth chapter of *De Interpretatione*:

$L(p$ or non-$p) \equiv M(p$ and non-$p)$,

where $L(p$ or non-$p)$ doesn't imply Lp or $L\sim p$, and $M(p$ and $\sim p$ doesn't imply Mp and $\sim M\sim p$, or $\sim Mp$ and $M\sim p$.

We can say yet that for Aristotle, in this moment possibility is only what we name contigency, and there is no pragmatic advantage to consider in its extension the strict necessity. Two and two were four yesterday, are four today and will be four tomorrow, by necessity. There is no sense to speak about the possibility of two and two tomorrow, or at least, no pragmatical advantage. Then, necessity is not possible.

The other question, and so interesting, is the fact that Aristotle speaks about two possibilities, as there are two necessities, absolute and hypothetical. The two possibilities are the following: possibility *stricto sensu* (*dynaton*) and admissibility (*endexomenon*), the second is only subjective. They

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[20]I would prefer capital letters to lowercase because the dictum looks more like a predicate than a proposition.

are one in front another, as the second, admissibility is "antistrophic" concerning the first or strict possibility, which is about reality (22a 15–16). Both possibilities have the same formal structure, but they have different denotations and senses.

References

1. Whitaker C. W. A. *Aristotle's* De Interpretatione*: Contradiction and Dialectic.* Oxford: Clarendon Press, 1997.

2. Васильев Н. А. Воображаемая (неаристотелева) логика. *Журнал министерства просвещения*, 1912, август, с. 207–246.